Sequential Search with Flexible Information *

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April 2024

Abstract

We consider a model of sequential search in which a decision-maker (DM) has to choose one alternative from a fixed set. All available alternatives are iid random variables and ex-ante unknown to the DM. Before making a choice, contrary to the standard search literature, we allow the DM to decide how much and what kind of information to acquire about each alternative, e.g., design different job market interviews for candidates with different arrival ranks. We find that optimal interviews have an intuitive property – the first arriving candidates are treated harshly, and their interviews are harder to pass, while later candidates' interviews are easier to pass. We compare the unconditional probabilities of choice and study the discrimination the order of inspection can cause. We argue that discrimination is sensitive to the functional form of the cost of learning. We consider several extensions, and we show that it may be optimal for the DM to interview an inferior candidate first and that a naive affirmative action policy can increase discrimination.

JEL-codes: D81, D83, D91, J71.

Keywords: sequential search, rational inattention, discrete choice.

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*We are grateful to Yair Antler, Sarah Auster, Francesc Dilmé, Duarte Gonçalves, Pavel Kocourek, Eugen Kovac, Filip Matějka, Fabio Michelucci, José L. Moraga, Rastislav Rehák, Max Riegel, Nicolas Schutz, Rava da Silveira, Yiman Sun, David Ronayne, Mark Whitmeyer and seminar participants at EPFL, CERGE-EI, Basel U, as well as the participants of The 12th Israeli Game Theory Conference, 13th Conference on Economic Design, Oligo Workshop 2023, EEA-ESEM 2023 conferences, MaCCI annual 2024, for helpful comments and valuable feedback. Funding by the German Research Foundation (DFG) through CRC TR 224 (project B03) is gratefully acknowledged.

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1 Introduction

Job market interviews are an integral part of hiring decisions. The usual setting includes a (female) employer who considers a fixed set of a priori identical (male) candidates whose suitability for the job is ex ante uncertain. The employer acquires costly information on the candidates through individual interviews which are conducted sequentially. At any point in time, the employer can stop her search and hire one of the candidates. The general problem was originally introduced in the seminal paper of Weitzman (1979), and since then it has been known as the "Pandora's Boxes Problem". Within this literature, the obvious question is to characterize the employer's optimal interviewing strategy.

While different variants have been studied within both Economics and Computer Science (Beyhaghi and Cai, 2023), not much has been said on the specification with fully flexible information acquisition technology. This means that the existing literature exogenously restricts the set of available interviews that the employer can conduct. While one can imagine that this restriction is innocent in settings where the interviewing strategy is dictated by some exogenous institution (e.g., whenever hiring is based on standardized tests), in most applications of interest the interviewer is free to conduct any interview she deems appropriate. As a result, allowing for fully flexible information acquisition is not a merely technical extension, but rather a fundamental aspect of a reasonable model of sequential interviews.

On the other hand, in all fairness to the existing literature, the presence of exogenous restrictions on the set of feasible interviews is not really surprising given the complexity of the problem with flexible information acquisition. At the same time, the developments in the literature of rational inattention have provided us with new tools that allow us to revisit this problem and provide new insights on the structure of the optimal interviewing strategy.

Formally, in this paper, the type of each candidate is drawn independently from the same (Bernoulli) distribution. Of course, the actual types remain unobservable to the employer. The candidates arrive for interviews sequentially at a fixed order. An interview takes the form of a usual Bayesian signal that is chosen by the employer. Aligned with the rational inattention literature, the cost of each signal is posterior-separable (e.g., Caplin et al., 2022; Denti, 2022). Upon observing the realized signal of an interview, the employer may either reject the candidate and proceed to the next interview or hire him right away. In this sense, our model maintains the no-recall assumption, which is common in many papers in the literature on Committed Pandora's Boxes (Beyhaghi and Cai, 2023, Section 2.2) as well as in the literature on the secretary's problem (Correa et al., 2021, and references therein). This assumption is natural in settings where the candidates have outside options and/or big egos that do not allow them to consider employers that previously rejected them. In this sense, it fits well in job markets of highly-skilled/highly-reputed candidates.

Then, we proceed to characterize the optimal interviewing strategy by reducing the dynamic information acquisition problem into a static one. Most interestingly, our result uncovers a special feature of the optimal interviewing strategy. Accordingly, we show that candidates that are interviewed earlier face a "more difficult interview", in that the expected quality of an earlier candidate who passes (resp., who fails) the test is larger than the expected quality of a later candidate who passes (resp., who fails) the test. This means that the optimal signals are ranked with respect to a new stochastic order. Remarkably, this order turns out to have striking similarities with the likelihood ratio dominance order for lotteries (Shaked and Shanthikumar, 2007, and references therein) and the information bias order (Gentzkow et al., 2014; Charness et al., 2021).

The implications of the previous result is twofold. First, the employer has the luxury to overshoot for very high expected quality in early candidates, as there is still plenty of candidates to come. At the same time, by having a large expected quality of a failed early candidate, the employer makes the interview of this early candidate relatively cheap, thus balancing the risk that she undertakes (via overshooting) and the information acquisition cost that she has to incur.

From the candidates' point of view, this strategy induces a tradeoff: early candidates have a lower probability of being hired conditional on being interviewed, but they also have a higher probability of being interviewed in the first place. Interestingly, it is not the case that one of the two effects always overtakes the other, meaning that the total probability of being hired is not monotonic with respect to the order in which candidates are scheduled to be interviewed. Our results shed light on a possible source of discrimination: primacy (recency) effects, when options that are presented earlier (later) are more likely to be chosen. We show that both effects can be present when the manager is rationally inattentive. Results depend on the non-trivial properties of the cost function of the information acquisition.

Additionally, we consider several simple extensions of our model. First, we relax iid assumption and show that flexible information technology can generate unexpected predictions in the search environment. In particular, if the candidates are a priori different, it may be optimal for the manager to consider a priori worse candidate first. Second, we discuss a naive policy that forbids discrimination in the interview process: we force the manager to choose the same interview for all candidates. We show that such restriction may increase the discrimination that the sequential search process can cause. Intuitively, in this case, the manager wants to bear less risk and conduct easier interviews, which always favors the first candidates.

Our results are important for several strands of literature. First, our work should be primarily seen as part of the literature on Pandora's Boxes. For an excellent recent overview of this literature, we refer to Beyhaghi and Cai (2023). Within this literature, particularly close to our paper is the stream that assumes no recall, known as the Committed Pandora's Box Problem. However, as we have already mentioned, ours is the first paper to consider full flexibility in information acquisition, thus opening a new avenue of research within this literature.

Second, our work can be seen as part of a broad stream within the dynamic rational inattention literature that focuses on the timing of information acquisition (Steiner et al., 2017; Morris and Strack, 2019; Zhong, 2022; Hebert and Woodford, 2023). There is variation in the underlying assumptions that they impose, e.g., some allowing for flexible information acquisition, some asumming discounting, some considering continuous time. At the same time, all these papers differ from ours in that they allow for information acquisition about the entire state space at any point in time. In our context this would imply that the employer can potentially interview multiple candidates at any point in time, and may also invite back candidates for follow-up interviews. While this assumption certainly makes sense for the applications that these authors have in mind, it seems less appealing in the job market setting that we have in mind as our main application.

Third, our paper is incidentally related to the literature on ordering Bayesian signals, by introducing our difficulty order. Most of the existing literature focuses on ranking signals with respect to informativeness (Blackwell, 1951). More recently, in an attempt to model information biases, Gentzkow et al. (2014) introduced a new partial order which is very similar in various aspects to our difficulty order.

Finally, there is a strand of literature on ordered consumer search, studying dynamic information acquisition about different consumption choices before an eventual purchase decision is made (for an overview, see Armstrong, 2017). The main difference to our work is that the focus of this literature is on the role of different asymmetries across choices, e.g., which product does the consumer inspect first when the products differ in price or differ in inspection costs? Similarly to the literature on Pandora's Boxes, most of the work on ordered consumer search imposes strict exogenous assumptions on the information acquisition technology, which at the outset seems quite natural in the context of the corresponding applications.

2 Model

We study a (female) employer who considers an ordered set of a priori identical (male) candidates $I = \{1, \dots, T\}$. Each candidate $i \in I$ is associated with a type

$$\theta_i \in \Theta := \{\text{Good}, \text{ Bad}\} = \{G, B\},$$

which is independently drawn from the same distribution that assigns probability $\mu \in (0, 1)$ to the good[1] type G.

The employer must choose one candidate, and there is no outside option. Before making a decision, she may acquire information about the candidates' types. Information acquisition is sequential, following the candidates' order. That is, at stage i, the employer selects a Blackwell experiment $\sigma_i : \Theta \to \Delta(S_i)$ for candidate i. Upon observing a signal realization $s \in S_i$, she forms a posterior distribution on Θ, that we identify with her posterior belief about the state G

$$p_i := \frac{\mu\sigma_i(s|G)}{\mu\sigma_i(s|G) + (1-\mu)\sigma_i(s|B)}.$$

It follows from the work of Kamenica and Gentzkow (2011) that each experiment is identified by a mean-preserving distribution of posteriors, i.e., by some $\pi_i \in \Delta([0, 1])$ such that $\mathbb{E}_{\pi_i}(p) = \mu$. A fully uninformative signal is one that puts probability 1 to the prior μ.

[1]The binary type assumption is not essential for our analysis. Our results hold in a more general setting, more specifically, when the utility of the manager only depends on the beliefs about posterior means as is typically assumed in the literature of the information design or costly information acquisition, for example, see Arieli et al. (2023) and Mensch and Malik (2023) for the recent references. We hold the binary type assumption mainly for the simplicity of the exposition.

After having updated to belief p_i^s about candidate i, the employer either hires i or proceeds to interview the next candidate $i+1$. We assume no recall, i.e., if a candidate is not hired right after an interview, he is no longer available to the employer.[2] Thus, the employer chooses an action

$$a_i \in A_i := \{0, 1\}$$

following the realization of an interview for candidate i, where action 0 corresponds to not hiring a candidate and 1 – to hiring. Hiring a good candidate brings utility 1 and hiring a bad candidate – 0.

Formally, a non-terminal history at round $i \in \{1, 2, \ldots, T\}$ is identified by the set of realized posteriors for all candidates $j \in \{1, \ldots, i-1\}$, i.e.,

$$\mathcal{H}_i := [0, 1]^{i-1}.$$

The employer's action at every $h \in \mathcal{H}_i$, consists of a signal, that leads to a posteriors π_i^h and a mapping $\alpha_i^h : \text{supp}(\pi_i^h) \to A_i$. Whenever, $p_i \in \text{supp}(\pi_i^h)$ is realized and $\alpha_i^h(p_i) = 1$ is chosen, a terminal history is reached, and candidate i is hired. In case round T is reached, the last candidate will be hired regardless of the realization of the respective signal. A typical strategy of the employer is henceforth denoted by $(\boldsymbol{\pi}, \boldsymbol{\alpha})$.

Information acquisition is costly. In line with the rational inattention literature we consider posterior separable costs (Caplin et al., 2022): signal π_i costs

$$C(\pi_i) = \lambda \mathbb{E}_{\pi_i}[c(p_i)], \tag{1}$$

where $\lambda \in \mathbb{R}_{++}$ is the marginal cost of information and $c : [0, 1] \to \mathbb{R}$ is continuous, strictly convex and smooth on the interior of the unit interval, and $C(\mu) = 0$ (the cost of fully uninformative signal equals to 0). The most common such specification is the case when c is the negative Shannon entropy.

If the employer chooses the action (π_i^h, α_i^h) at history $h \in \mathcal{H}_i$, her expected payoff is equal to

$$\mathbb{E}_{\pi_i^h}\Big[\alpha_i^h(p_i)p_i + (1 - \alpha_i^h(p_i))V_i - \lambda c(p_i)\Big],$$

with V_i denoting her maximum net expected payoff in case she continues and interviews candidate $i+1$. Note that V_i depends only on the number of remaining candidates, as the types of the different candidates are drawn independently from the same probability distribution, and there is no recall possibility. Hence, without loss of generality, we can restrict attention to $\mathcal{H}_i$-measurable strategies, i.e., to strategies such that $(\pi_i^h, \alpha_i^h) = (\pi_i, \alpha_i)$ for all $h \in \mathcal{H}_i$. This means that the employer's expected payoff is simplified to

$$\mathbb{E}_{\pi_i}\Big[\alpha_i(p_i)p_i + (1 - \alpha_i(p_i))V_i - \lambda c(p_i)\Big]. \tag{2}$$

[2] There can be several rationales for such assumption, e.g., it is caused by psychological factors of the rejected agent (pride, etc.), or of the employer (extreme case of limited memory), or by conditions on the labor market (other firms immediately hire rejected candidate).

Definition 1. *The full dynamic problem of the manager is to find* $(\boldsymbol{\pi}, \boldsymbol{\alpha})$ *such that*

$$
\begin{aligned}
(\boldsymbol{\pi}, \boldsymbol{\alpha}) &\in \left(\arg\max_{(\pi_i,\alpha_i)} \mathbb{E}_{\pi_i}\Big[\alpha_i(p_i)p_i + (1-\alpha_i(p_i))V_i - \lambda c(p_i)\Big]\right) \quad s.t. \qquad (3) \\
V_i &= \max_{(\pi_{i+1},\alpha_{i+1})} \mathbb{E}_{\pi_{i+1}}\Big[\alpha_{i+1}(p_{i+1})p_{i+1} + (1-\alpha_{i+1}(p_{i+1}))V_{i+1} - \lambda c(p_{i+1})\Big], \qquad (4) \\
V_T &= 0. \qquad (5)
\end{aligned}
$$

Constraint (4) ensures dynamic consistency, that is DM behaves optimally in every history. Constraint (5) captures the intuition that if the final candidate is indeed reached, it implies that the DM has rejected all candidates before. In that case DM rejects all candidates and ends up with zero payoff.

3 Optimal interviewing strategy

The optimal strategy $(\boldsymbol{\pi}, \boldsymbol{\alpha})$ in problem (3) is the collection of the optimal actions (π_i, α_i) for all $i \in I$. Note that the interview design problems at some stages i, j differ only by their continuation values V_i, V_j. Therefore, in the dynamic problem, the employer behaves as if she solves a collection of static problems with different continuation values. These continuation values are determined from the future behavior of the employer, and the value is *exogenous* at stage i. Thus, at stage i, a continuation value V_i serves the role of an outside option to the employer. Therefore, we conclude that at each stage i, the employer solves a static problem with an exogenous outside option. A static problem with an exogenous outside option is a building block for the dynamic problem, and we discuss a static problem in great detail in this section.[3]

Additionally, we observe that at stage i given the realized value p_i the employer simply selects candidate i if $p_i \geq V_i$ and continues search otherwise, and therefore in our previously-stated optimization problem we can replace $\alpha_i(p_i)p_i + (1-\alpha_i(p_i))V_i$ with $\max\{p_i, V_i\}$. Thus, the optimization problem at stage i boils down to the following (static) optimization problem with parameter $V := V_i$.

Definition 2. *The static problem (or the problem with exogenous outside option) is*

$$
\max_{\pi} \mathbb{E}_{\pi}\Big[\underbrace{\max\{p, V\} - \lambda c(p)}_{\phi(p,V,\lambda)}\Big]. \qquad (6)
$$

In the analysis we only consider the scenarios when the employer's learning strategy is interior, that is, the domain of her optimal signal π does not include 0 and 1. For that, we use mild boundary conditions for the cost function that are trivially satisfied, for example, when c is negative entropy.

[3] Such a problem is a variant of the problem of a rationally inattentive agent with an exogenous outside option, see, e.g., Matějka and McKay (2015), Caplin and Dean (2013) for the reference.

Assumption 1.

$$\lim_{p \to 0^-} c'(p) < c'(\mu) - \frac{1}{\lambda},$$
$$\lim_{p \to 1^-} c'(p) > c'(\mu) + \frac{1}{\lambda}$$

We discuss properties of the static problem using two technical lemmas. The first lemma characterizes the solution exploiting the convexity and differentiability of the function $c(p)$.

Lemma 1. *Let the function c satisfy Assumption 1, then the following statements hold:*

1. *The solution to problem (6) exists and is unique.*
2. *There exist two thresholds $V_L, V_H \in (0,1)$ with $V_L < \mu < V_H$, such that the optimal signal π_V satisfies:*

$$\begin{array}{lcl} V \leq V_L & \Rightarrow & \text{supp}(\pi_V) = \{\mu\}, \\ V_L < V < V_H & \Rightarrow & \text{supp}(\pi_V) = \{p_V^L, p_V^H\}, \\ V \geq V_H & \Rightarrow & \text{supp}(\pi_V) = \{\mu\}. \end{array}$$

3. *The optimal hiring decision is given by the following:*

$$\begin{array}{lcccl} V \leq V_L & & & \Rightarrow & \alpha(\mu) = 1, \\ V_L < V < V_H & \textit{and} & p = p_V^H & \Rightarrow & \alpha(p) = 1, \\ V_L < V < V_H & \textit{and} & p = p_V^L & \Rightarrow & \alpha(p) = 0, \\ V \geq V_H & & & \Rightarrow & \alpha(\mu) = 0. \end{array}$$

The previous lemma is illustrated in Figure 1 below. The idea is that $\phi(p, V, \lambda)$ consists of two strictly concave parts, with a kink at V. This induces the two posteriors p_V^L and p_V^H, and as the prior lies between these two the employer will acquire an informative signal that distributes its probability to these two posteriors; otherwise, she will pick the completely uninformative signal. These posteriors are obtained, for example, using the concavification technique as in Caplin and Dean (2013).[4]

The key observation is that both p_V^L and p_V^H are continuously increasing in V (see Lemma A3 in the Appendix for a formal proof). Moreover, we have

$$\lim_{V \to 0^+} p_V^L = \lim_{V \to 0^+} p_V^H = 0 \text{ and } \lim_{V \to 1^-} p_V^L = \lim_{V \to 1-} p_V^H = 1.$$

Hence, for sufficiently large V, the whole interval $[p_V^L, p_V^H]$ will lie to the right of μ. Likewise for sufficiently small V, the interval will lie to the left of μ. Thus, we can define the two thresholds:

$$\begin{array}{rcl} V_H & := & \min\{V \in [0,1] : p_V^L \geq \mu\}, \\ V_L & := & \max\{V \in [0,1] : p_V^H \leq \mu\}. \end{array}$$

[4]For recent use of concavification to the related rationally inattentive problems, see, e.g., Jain and Whitmeyer (2021), Kim et al. (2022).

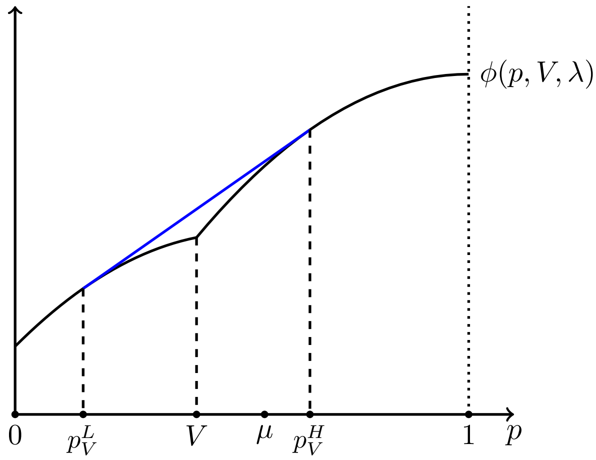

Figure 1: The employer's static payoff function and its concavification when $V \in (V_L, V_H)$.

Note that by the prior $\mu \in (0,1)$ being full-support, we obtain both $V_L \in (0,1)$ and $V_H \in (0,1)$.

In the dynamic problem of the employer, an outside option at stage i equals the value of the problem at stage $i+1$. In its turn, this value equals the maximal attained value in a static problem (6) for a particular value of outside option V. To study the maximal attained value in static problem we define a value function $g : [0,1] \to [0,1]$ such that

$$g(V) = \max_{\pi} \mathbb{E}_{\pi}\Big[\phi(p, V, \lambda)\Big].$$

Function $g(V)$ is clearly non-decreasing by construction, as $\phi(p, V, \lambda)$ is weakly increasing in V for every p. We note that function $g(V)$ is linear on $[0, V_L]$ and $[V_H, 1]$, viz., we have $g(V) = \mu$ if $V \le V_L$ and $g(V) = V$ if $V \ge V_H$, as in either of these two regions the employer does not incur any costs for acquiring information, and makes a hiring decision straight away. The following lemma completes the analysis for the entire unit interval.

Lemma 2. *The function g is strictly increasing, convex and differentiable everywhere in $[0,1]$.*

We will now proceed to characterize the solution to the dynamic problem. To do so, we first define a specific sequence of static problems, by means of a sequence of outside options, viz., for each $i \in \{1, \ldots, T\}$, we have

$$V_i := g(V_{i-1}), \tag{7}$$

with $V_T = 0$.

Lemma 3. *For the sequence $(V_i)_{i=1}^T$ defined in (7), the following statements hold:*

1. *The outside option V_i is strictly decreasing in i.*

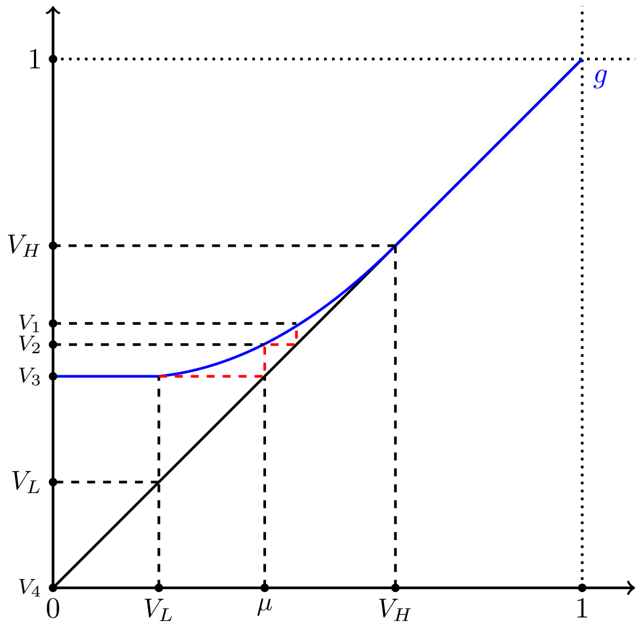

Figure 2: The sequence of outside options with four candidates.

2. For every $i \in \{1, \ldots, T-1\}$, we have $V_L < V_i < V_H$.

Let us illustrate Lemma 3 graphically (Figure 2). First of all, by combining convexity and differentiability of g with the fact that $g(V) = V$ for all $V \geq V_H$, it follows that $g(V) > V$ for all $V < V_H$. Hence, V_i keeps shrinking as the employer moves to later candidates. This is not surprising, as there are fewer candidates left to interview. The fact that for all candidates except the last one the outside option lies always in the learning region (V_L, V_H) holds similarly.

Theorem 1. *The solution to the dynamic problem of Definition 1 is as follows:*

1. *At every round $i \in \{1, \ldots, T-1\}$, the manager draws the signal π_{V_i} which is optimal in the static problem (according to Lemma 1), with the outside option V_i that we defined in (7). Moreover, we have:*
   - *(a) If $p^H_{V_i}$ is realized, the search stops and candidate i is hired.*
   - *(b) If $p^L_{V_i}$ is realized, the search continues to candidate $i+1$.*
2. *At round T, the manager does not acquire information and hires the candidate right away.*

The optimal interview design of the manager in the dynamic problem is very straightforward: she continues the search until she receives a high signal about the quality of a candidate. If only low signals have been realized during the first $T-1$ interviews, she simply chooses the last candidate with the fully uninformative signal. We employ the natural interpretation of an optimal interview at stage i as a binary test. The manager offers a test to a candidate i. If a candidate passes, he is hired; if a candidate fails, he is discarded. We state our main result about the characterization of the optimal tests in the next section.

4 Difficulty of interviews

To compare the different tests that the manager (optimally) chooses for the different candidates, we introduce the following partial order.

Definition 3. *Let π_i and and π_j be two binary signals, with $p_i^L < p_i^H$ and $p_j^L < p_j^H$ being the posteriors beliefs in the respective supports. We say that π_i is more difficult than π_j if*

$$p_i^L > p_j^L \text{ and } p_i^H > p_j^H.$$

The condition above has a simple interpretation. The two candidates are ex ante identical from the point of view of the employer, and they are offered one test each such that, whenever there is a tie, i is deemed better than j, i.e., in particular,

(a) if both of them pass their respective tests, the expected quality of i is higher than the expected quality of j, and

(b) if both of them fail their respective tests, the expected quality of i is higher than the expected quality of j.

Recall that the two signals π_i and π_j are respectively characterized by the underlying experiments σ_i and σ_j (see Section 2).

Definition 4. *We say that experiment σ_i likelihood-ratio dominates experiment σ_j whenever, for every signal realization $s \in \{H, L\}$,*

$$\frac{\sigma_i(s|G)}{\sigma_i(s|B)} > \frac{\sigma_j(s|G)}{\sigma_j(s|B)}.$$

The underlying idea is as follows: conditional on every test result, the relative evidence for the good type is stronger under i's interview than under j's interview. Note that our notion of likelihood ratio dominance bears similarities with the one that is often used in the literature to compare lotteries (Shaked and Shanthikumar, 2007). A similar relation has been used in the literature on biased information sources (Gentzkow et al., 2014; Charness et al., 2021). Remarkably, our difficulty order will be characterized in terms of likelihood ratios.

Proposition 1. *The following are equivalent:*

(i) Signal π_i is more difficult than signal π_j.

(ii) Experiment σ_i likelihood-ratio dominates experiment σ_j.

(iii) For every prior μ, the passing probability under σ_i is lower than the passing probability under σ_j, i.e.,

$$\underbrace{\mu\sigma_i(H|G) + (1-\mu)\sigma_i(H|B)}_{\textit{passing probability under } \sigma_i} < \underbrace{\mu\sigma_j(H|G) + (1-\mu)\sigma_j(H|B)}_{\textit{passing probability under } \sigma_j}.$$

From the previous result it follows directly that our notion of more difficult interview does not depend on the prior. This is a desirable property, satisfied by other well-known orders over the set of Bayesian experiments. The idea is that difficulty is a property of the test alone, defined independently of the candidate who takes the test.

Furthermore, if the two candidates actually share the same prior, i's interview is harder than j's interview if and only if the probability to pass the test is lower for i than it is for j.

Let us now state our main result which says that, in the optimal interviewing strategy, the interviews decrease in difficulty as the employer proceeds to later candidates.

Theorem 2. *In the optimal strategy from Theorem 1, the following hold:*

1. *Difficulty is decreasing with respect to the order of being interviewed, i.e., for all $i \in \{1, \ldots, T-2\}$, signal π_{V_i} is more difficult than signal $\pi_{V_{i+1}}$.*
2. *As the number of candidates grows large, we obtain:*

$$\lim_{T\to\infty} p^L_{V_1} = \mu \text{ and } \lim_{T\to\infty} p^H_{V_1} = V_H.$$

Hence, the probability of the first candidate being hired converges to 0.

In Figure 3, we show an example of an optimal learning strategy. On the vertical axis we have the number of remaining candidates, besides the one currently interviewed. So for instance, if there are ten candidates in total, we depict the optimal interviews for the first nine, recalling that the last one will be hired anyway without an interview.

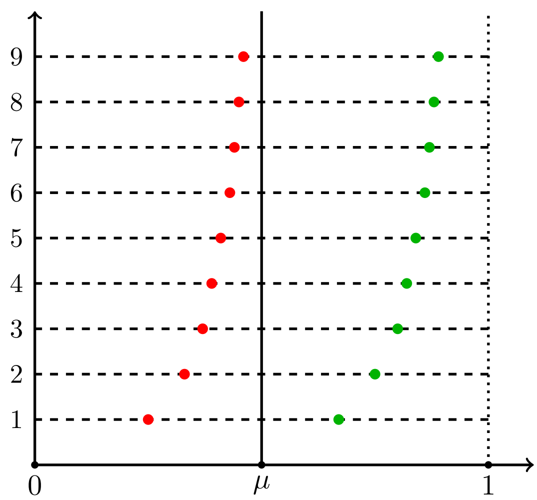

Figure 3: Optimal interviews as a function of the number of (additional) remaining candidates.

Decreasing the high posterior realizations $p^H_{V_i}$ is intuitive. If a posterior $p^H_{V_i}$ is realized on the interview i, the employer stops the search and chooses candidate

i. Thus, it makes sense that in order for the employer to stop the search early, she needs to be sufficiently certain that the candidate she hires is good, as she is foregoing the chance to interview many other potential candidates.

Decreasing the low posterior realizations $p^L_{V_i}$ is less intuitive. By using such a strategy, the employer optimally procrastinates: instead of acquiring the most information during the first interviews, she wants to spread expected information acquisition towards all interviews. Intuitively, during the first interviews, she offers hard tests for the applicants because she has some applicants that are left. The employer wants to bear the risk and try to "catch a big fish" at the beginning. The fewer candidates are left, the safer the strategy used by the employer is.

Additionally, we describe the dynamics of the optimal interviews in terms of the statistical errors that the employer makes. We consider type I and type II errors as the probability of hiring a bad candidate, and as the probability of rejecting a good candidate, respectively. Combining results from Theorem 2 and Proposition 1, we obtain that in the optimum, the sequence of type I errors decreases in i, and the sequence of type II errors increases in i. At the first stages, the employer bears the risks, offers the hardest tests, and tolerates the false negatives, whereas in the later stages she plays safer, decreasing the probability of false negatives and increasing the probability of false positives.

5 Discrimination

From Theorem 2, we know that the optimal interviews are more difficult for earlier candidates: *conditional* on being interviewed, later candidates get an easier test than earlier ones. However, from Theorem 1, we know that the probability that a candidate is interviewed at all is lower than that of his predecessors, since it depends on those candidates failing their interviews. The aggregate effect is unclear. In this section, we ask which effect dominates, that is, what can we say about the *unconditional* probability of a candidate being hired? We show that the answer to this question is sensitive to assumptions on the cost function, c, the marginal cost λ, and the prior belief μ. In what follows, we first focus on the case of two candidates and provide sufficient conditions under which the first or the second candidate may be discriminated against. These conditions are general enough to incorporate some commonly used cost functions such as entropy and quadratic forms. Finally, we evaluate the choice probabilities for a general T assuming quadratic costs.

Denote by q_{iT} the unconditional probability of candidate i being accepted from a pool of T candidates. An interview design is *discriminatory* if these choice probabilities are not uniform even though candidates are a priori identical. The only difference in the candidates, insofar as the employer is concerned, is their relative position in the interview.

5.1 Two candidates

In order to make the problem tractable, we make certain assumptions on the cost function. We first define symmetric cost functions.

Definition 5. *c is said to be symmetric about a point $z \in (0,1)$ if for any pair $p, q \in [0,1]$*

$$|z-p| = |z-q| \implies c(p) = c(q)$$

Note here that the point of symmetry, z may depend on the prior as in quadratic functions or may be independent of the prior as is the case of entropy. For example, for the quadratic cost function $c(p) = (p-\mu)^2$ the axis of symmetry is $z = \mu$, for the entropic cost $c(p) = p\log p + (1-p)\log(1-p)$ the axis of symmetry is $z = \frac{1}{2}$. We denote the subset of posterior separable and symmetric cost functions by $\mathcal{C}_s$. Moreover, we assume that:

Assumption 2. *c belongs to either of the two following families:*

$$\mathcal{C}_1 = \left\{c \in \mathcal{C}_s \,\middle|\, c' \text{ is concave on } (0,z)\right\}$$

$$\mathcal{C}_2 = \left\{c \in \mathcal{C}_s \,\middle|\, c' \text{ is convex on } (0,z)\right\}$$

We say that a candidate 1 is *favored* if the unconditional probability of hiring him strictly exceeds 0.5.

Proposition 2. *Under Assumption 2, we have:*

1. *If $\mu = z$ or $c \in \mathcal{C}_1 \cap \mathcal{C}_2$, no candidate is favored.*

2. *If $c \in \mathcal{C}_1$, there exists an open neighborhood of z denoted by B_z such that candidate 1 is favored whenever $\mu \in B_z \cap (z,1)$ and candidate 2 is favored whenever $\mu \in B_z \cap (0,z)$.*

3. *If $c \in \mathcal{C}_2$, there exists an open neighborhood of z denoted by B_z such that candidate 1 is favored whenever $\mu \in B_z \cap (0,z)$ and candidate 2 is favored whenever $\mu \in B_z \cap (z,1)$.*

Proposition 2 suggests that the probability of choice depends on the parameters of the model and properties of the cost function in a very nontrivial way, even when $T = 2$. In particular, it depends on a combination of the curvature of the first derivative of c and the prior belief. To get some intuition for the result, we point out that if $\mu = z$, then the rates of change of the posteriors, p_μ^H and p_μ^L, with respect to μ are identical. Consequently, the comparison between q_{12} and q_{22} depends on the absolute value of the derivative of the high posterior p_μ^H. In its turn, the latter depends on the sign of the third derivative of c.

For the case of entropic costs, the closed form solution for the posteriors helps in arriving at a more global property which is summarized in the following lemma.

Proposition 3. *Let c be negative entropy. Then, the first candidate is favored if and only if $\mu \in \left(\frac{1}{2}, 1\right)$.*

In that case, the employer favors the first candidate in the *lemon-dropping* market ($\mu > 0.5$) and favors the second candidate in the *cherry-picking* market ($\mu < 0.5$). Bartoš et al. (2016) find that endogenous attention leads to discrimination when candidates have different expected productivity. Our result suggests that discrimination may present if the candidates are ex-ante the same, but the choice problem is sequential.

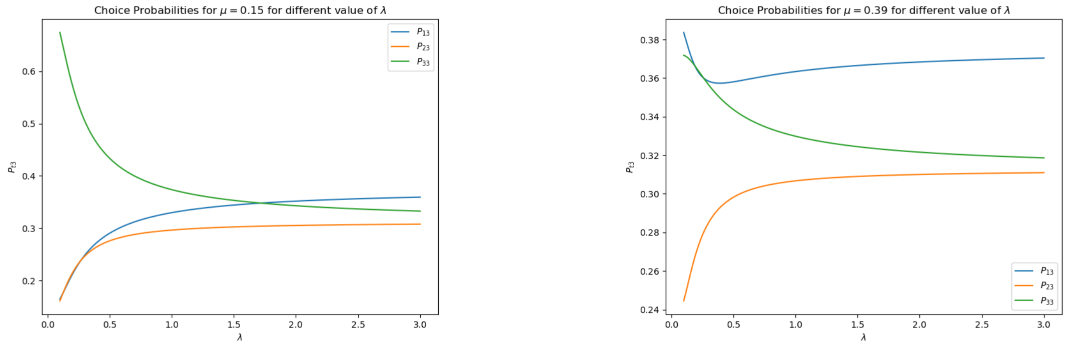

Figure 4: *Left*: non-monotonicity between q_{13} and q_{23}. *Right*: non-monotonicity between q_{13} and q_{33}.

5.2 Three or more candidates

In a very special case, when $c'(p)$ is linear, the manager's behavior does not depend on the expected productivity of a candidate. We show that such behavior is a general feature of the problem in this special case. The sequence q_{iT} behavior for a given T does not depend on the expected productivity μ. In particular, we show in the Proposition below that q_{iT} is strictly decreasing in i for all $i < T-1$. It is better to be first: the candidates in the beginning have a relative advantage for being hired.

Proposition 4. *Let $c(p) = (p-\mu)^2$. Then for any fixed T unconditional probability to choose candidate t, q_{iT} is strictly decreasing in t for all $t < T-1$ with $q_{T-1T} = q_{TT}$.*

We argue that such a general result about the monotonicity of q_{tT} is very sensitive to the cost specification. Moreover, in general, when $T > 2$, endogeneity of the continuation values V_i generates an additional effect on the sequence q_{iT}. In order to provide examples when non-trivial trade-offs may arise, we present an analysis for some cases for $T = 3$ and entropic cost. In particular, we consider how the probabilities q_{i3} change with respect to the marginal cost of information λ. For example, consider the behavior of q_{13} with respect to λ. We can write that $dq_{13}/d\lambda = \partial q_{13}/\partial\lambda + (\partial q_{13}/\partial V_1) \times (\partial V_1/\partial\lambda)$. In general, the sign of $dq_{13}/d\lambda$ is ambiguous: because of the inequality $V_1 > \mu$, the inequality $\partial q_{13}/\partial\lambda < 0$ holds but also $\partial q_{13}/\partial V_1 \times \partial V_1/\lambda > 0$ holds because both terms in the product are negative.

On Figure 4, we provide examples of non-monotonic behavior of choice probabilities q_{iT}. Moreover, we show that the order of probabilities also may change with respect to λ. On the left figure, we consider the case with small ex-ante productivity $\mu = 0.14$. If information is almost free, the inequality $q_{13} > q_{23}$ clearly holds, because $0.15 > 0.85 \times 0.15$ holds. For greater values of λ, the first interview is becoming easier to pass and, therefore, q_{13} increases. However, q_{23} also increases and, moreover, $q_{23} > q_{13}$ that suggests that because the information becomes getting expensive, the employer simplifies the second interview even more. If λ becomes larger, the inequality $q_{13} > q_{23}$ again holds, suggesting that the employer chooses a more similar level of difficulty on the first and the second interviews.

On the right figure, we consider the case with $\mu = 0.39$. Contrary to the previous example, if the information becomes slightly expensive, q_{13} decreases. For some values of λ, the first interview becomes relatively hard because the function q_{13} is steep. This effect leads to the inequality $q_{33} > q_{13}$. However, when the information becomes expensive, the first interview becomes easier, and inequality $q_{13} > q_{33}$ starts to hold again.

If λ becomes large, the behavior of the sequence q_{i3} becomes monotonic irrespectively of the value of μ. Next proposition states the result that is similar to the Proposition 4 for the entropic cost when the marginal cost of information is very large.

Proposition 5. *Consider $T = 3$ and assume the entropic cost of information. If λ goes to ∞ then q_{13} converges to $\frac{3}{8}$, and both q_{23}, q_{33} converge to $\frac{5}{16}$.*

The Proposition 5 has the following interpretation in terms of discrimination.

Corollary 1. *In the setup of Proposition 5 for any $\mu \in (0, 1)$ there exists such λ_μ that if $\lambda > \lambda_\mu$, then $q_{13} > q_{23}$ and $q_{13} > q_{33}$.*

Proposition 5 and Corollary 1 suggest that if information becomes expensive, it is better to be first. Perhaps surprisingly, the probabilities of choice are not uniform in the limit. Even if the information is very expensive, the employer learns during the first and the second interviews and chooses the first candidate with the highest probability. We leave the investigation of the behavior of sequence q_{iT} for larger values of T and nonquadratic cost for future research.

6 Different productivities

Throughout the text, we assumed that our candidates were identically and independently distributed. Although that may seem a substantial simplifying assumption, that assumption captures our initial motivation to investigate the role of endogenous information in the sequential dynamic problem. By independence, we shut down the learning motive that will generate an additional incentive to acquire the information at the first stages. The same distribution assumption allows us to isolate the sequential role of the decision. This assumption is less restrictive in its nature, and we relax it in this Section.

Our analysis are a reduced comparative statics exercise with respect to the prior belief about a candidate's productivity. In particular, we consider the situation in which one of the candidates is ex-ante marginally better than others. We investigate how such a perturbation influences the optimal structure of the interviews and comment on how it may impact discrimination. When the candidates have different expected qualities, the manager has an additional control variable – the optimal order of candidates. We analyze such a problem for the case with two available candidates at the end of the Section. Information about the expected qualities is public knowledge, and the optimal behavior of the manager generates incentives for the candidates to take particular interview slots. Therefore, the complete and careful investigation should reflect the candidates' responses on the optimal interview design and include the game-theoretic analysis. Such an analysis is beyond the scope of the current paper and the natural direction for future research.

We start with the scenario in which one of the candidates is better than the others. The case in which a candidate is worse can be analyzed identically. We assume that there is a candidate i' such that $\mu_{i'} > \mu$, where $\mu_i = \mu$ for all $i \neq i'$. We focus on a situation where the candidate i' is marginally better than others, $\mu_{i'} \approx \mu_i + \varepsilon$. In particular, such an assumption guarantees that the manager acquires information if she reaches any period $i < T$. For now, we assume that the order of the candidates is given, and the manager must interview the ex-ante best candidate at stage i'. We restrict our attention to uniformly posterior-separable cost functions in order to isolate effects of cheaper (more expensive) information about different candidates, i.e., c does not depend on μ in order to maintain the same information technology for all the candidates. The next Proposition identifies how the optimal structure of the interviews changes in this case.

Proposition 6. *Fix T and let exist $i' \leq T$ such that $\mu_{i'} > \mu_i$ for all $i' \neq i$ and $\mu_i = \mu$ for other i. Assume that if the manager reaches the period $i < T$, she acquires information. Solution to dynamic problem (1) changes as follows:*

1. *DM increases the difficulties of any interview i such that $i < i'$*
2. *DM does not change any interview i such that $i \geq i'$.*

Proposition 6 confirms the basic intuition about the role of the difficulty in the solution to the employer's problem. Because of non-recall property, a candidate i' makes no difference for the employer's problem for $i > i'$. The employer also chooses the same posterior beliefs interviewing the candidate i' – this is a standard property of the solution of the rational inattention problem with a uniformly posterior-separable cost function. However, the employer changes her behavior in all interviews i such that $i < i'$. Intuitively, she can risk more at the earlier stages because she expects a better candidate later. The employer chooses the harder interviews at the beginning, lowering the probability of success. However, if a candidate passes the interview, the employer hires, on average, a better candidate.

Although an inclusion of a better candidate has clear effect on the structure of the optimal solution, the impact on the unconditional probabilities of choice and, therefore, on the discrimination is generally unclear except some very special cases. Such an example is $i' = 1$. In this case, obviously, each q_{iT} for $i \geq 2$ is decreased by the same factor, and the sequence of q_{iT} has the same order in terms of inequalities. Below, we present two numerical examples in which the inclusion of the better candidates generates non-trivial spillover effects on the unconditional probabilities of choice and, hence, on discrimination.

Example 1. We consider the case $T = 3$ and $i' = 3$. In this case, both the first and the second interviews become harder. Therefore, the value q_{13} decreases. However, the behavior of the q_{23} is generally unclear. Although the probability of passing the interview for the second agent decreases, the employer interviews this candidate more often. Thus, in principle, the inclusion of the better rival may *increase* the chance of the candidate to be hired. Such a situation happens, for example, if the employer has the entropic cost of information and $\mu_3 = 0.61, \mu = 0.6, \lambda = 0.1$. That is, in this case, the change in the expected productivity of the third agent from 0.6 to 0.61 increases q_{23}.

Example 2. Yet another non-trivial impact of the inclusion of the better candidate is the effect on discrimination. We again consider the case $T = 3$ and $i' = 3$ in which the employer has an entropic cost. If $\mu_3 = \mu = 0.1, \lambda = 0.4$ then the inequality $q_{33} > q_{23} > q_{13}$ holds. The employer chooses the first candidate the least often. However, if $\mu_3 = 0.11$, the sign of the inequality changes. Both the first and the second interviews become harder, and for such parameters, both probabilities q_{13}, q_{23} decrease. However, these probabilities decrease disproportionally and the inequality $q_{33} > q_{13} > q_{23}$ holds. Therefore, the employer chooses the second candidate the least often in this case.

Another interesting observation concerns $q_{i'T}$, the unconditional probability of choosing the best ex-ante candidate. Although candidate i' is ex-ante the best, the employer may not choose him with the highest probability. Consider, for example, the case as in Section 5 in which $\mu_i = \mu$ for all i and the sequence q_{iT} is decreasing in i. By the continuity argument, if μ_T marginally increases, the sequence q_{iT} will still be monotone in i. Therefore, the employer will choose the best ex-ante candidate with the lowest probability. The dynamic effect dominates in the solution. The employer tries to save on the cost of information in the later stages and chooses worse candidates more often. Intuitively, this situation occurs because the employer can not control the order of inspection. In the following Proposition, we show that it is indeed the case. If the order of inspection is endogenous, the employer chooses the better agent more often. We state our result for the case $T = 2$.

Proposition 7. *Let function $c(p)$ belong to the class $\mathcal{C}_1$ from Assumption 2. Let $T = 2$ and suppose that candidate i is marginally better than the candidate j: $\mu_i \approx \mu_j + \varepsilon$. Let the employer solve Problem (1) and let her also choose the order of candidates. Then*

1. *If $\mu_j = z$ then the employer is indifferent in the order of candidates.*
2. *If $\mu_j > z$ then the employer interviews agent i the first and agent j the second.*
3. *If $\mu_j < z$ then the employer interviews agent j the first and agent i the second.*

In all cases inequality $q_{iT} > q_{jT}$ holds.

If the employer can choose the inspection order, she does it efficiently. Recall, that from Proposition 2 the inequality between q_{12} and q_{22} depends on the value of μ. Similarly, the employer decides whom to interview depending on the value of μ_j. In the «lemon-dropping» market, the employer interviews the better candidate; in the «cherry-picking» market, she interviews the worse candidate. In both cases, the optimal order increases the probability of hiring an ex-ante better candidate. Thus, similarly to the recent paper by Fosgerau et al. (2023), endogenous information amplifies the effect of the ex-ante differences in our model.

Interestingly, the results for the two candidates case suggests that the employer may want to start the search with the worst candidate and leave better candidates for later stages. This finding is a striking difference from results in the classical search literature, e.g., Weitzman (1979). In those models with restricted information structures DM always starts the search with the most promising alternative. Inspection of the worst candidate first can be a part of the solution in a more general model. For example, Doval (2018) finds such behavior in a version of the Pandora box model in which DM can choose an item without inspection. We provide another justification for such a finding based on the endogenous information channel.

7 Restricted interview design

At the optimum, the employer in our model fully leverages the flexibility of the interview design. She constructs different interviews depending on the serial position of a candidate. Utilizing the dynamic structure of the problem, the manager offers more difficult tests to the candidates who are arriving early. Such a difference in the treatment can create discrimination in the hiring outcomes, as discussed in Sections 5, 6, and may be seen as unfair. In this Section, we reduce the manager's power and restrict the feasible set of interviews. We consider the scenario in which the manager has to offer an interview of the same difficulty for all candidates.

We employ two interpretations of our restricted setting. In the first interpretation, the authority may ask an employer to design identical interviews for all candidates. We refer to this interpretation as the *structured interview* in the hiring process. A structured interview is usually understood as a series of predetermined questions the interviewer addresses to the applicants, evaluating their responses by a standardized procedure. In their guide to conducting a fair selection process, the National Institutes of Health, the primary agency of the United States government, suggests that using a structured interview reduces bias in the hiring procedure.[5] However, it is possible that such a recommendation is not directly applicable, for example, if tests become publicly available or if there is a possibility of information sharing between applicants, the authority may ask to design tests that are not identical, but have the same level of difficulty for all the candidates.

In this Section we consider a simplified version of the restricted interview design. The possible issue is that employer in the restricted case might be tempted to design similar interviews for the candidates but to take into account time of arrival for her hiring decisions. To counteract this, we assume that all candidates must have the same expected productivity level conditional on the employer's action. Independently from their serial numbers, rejected candidates have to have the same expected productivity. An acceptance belief also does not depend on the interview step.[6] Additionally, the procedure guarantees that, at the optimum, the manager chooses at most two posterior realizations. We formulate the restricted manager's problem as a restricted version of problem (1).

[5] Manager's Fair Selection Toolkit. Office of Equity, Diversity, and Inclusion, National Institutes of Health. https://www.edi.nih.gov/sites/default/files/public/EDI_Public_files/guidance/toolkits/managers/manager-fair-selection-toolkit01.pdf

[6] In order to not put our employer in risk of not hiring anyone, we assume that the last arriving candidate can be hired without interview.

Definition 6. *The restricted dynamic employer's problem is to find* (π, α) *such that*

$$\begin{aligned}(\pi, \alpha) &\in \left(\arg \max_{(\pi_{\text{supp}|\pi| \leq 2}, \alpha)} \mathbb{E}_{\pi}\Big[\alpha(p_i)p_i + (1-\alpha(p_i))V_i - \lambda c(p_i)\Big]\right) \quad s.t. \\ V_i &= \mathbb{E}_{\pi}\Big[\alpha(p_{i+1})p_{i+1} + (1-\alpha(p_{i+1}))V_{i+1} - \lambda c(p_{i+1})\Big], \\ V_{T-1} &= \mu, \\ V_T &= 0.\end{aligned} \tag{8}$$

We emphasize two differences between the restricted and the unrestricted problems. First, in the restricted problem, the employer chooses a posterior distribution only once. Therefore, we omit index i. Additionally, there is a restriction on the cardinality of the support of the posterior distribution.[7]

It is convenient to analyze the restricted problem (8) explicitly using the unconditional choice probabilities. The restricted problem is a special case of a static problem (2) with corresponding outside option. In Appendix B we reformulate the static problem (2) as an equivalent problem, in which maximization is over the unconditional choice probabilities only, and show that there is one-to-one mapping between unconditional choice probability and the optimal posteriors. Analogous analysis can be found in a recent paper by Fosgerau et al. (2023) and is similar to the expressing discrete static rational inattention problem as the log-sum as in Matějka and McKay (2015), Caplin et al. (2019).

Next proposition is our main result for this Section and it compares the solutions of the restricted and unrestricted problems.

Proposition 8. *Let* q^* *and* q_{1T} *be the optimal probability of hiring the first arriving candidate in the restricted and unrestricted problems with* T *candidates respectively. When* $T > 2$*, the inequality* $q_{12} > q^* > q_{1T}$ *holds.*

Proposition 8 identifies that in the restricted problem, employer chooses a test with intermediate difficulty: the test is easier than the hardest one, which is offered to the first candidate, and is harder than the easiest one, which is offered to the last interviewed candidate. Recall that the employer engages in more risky behavior in the unrestricted setting in the earlier stages. She chooses a test with a low probability of success because she can mitigate the failure in the future stages. Restriction on the interview design forbids such mitigation. The results in Proposition 8 are intuitive and naturally fit into the risk interpretation: the manager bears less risk in the earlier stages.

It is immediate from Proposition 8 that the restriction on interviews increases and decreases the probabilities of hiring the first and the last interviewed candidates, respectively. How does the restriction influence the chances of other candidates

[7] If the support of posterior distribution is not bounded, it is generally unclear whether Lemma 1 holds in the restricted setting. For example, anticipating the lack of choice in the following periods, DM may leverage her flexibility in the first period and design a test with more than two realizations. This strategy may allow the optimal dynamic behavior to be squeezed into a single test. Such a more general problem is out of this project's scope and is a natural direction for future research.

being hired is generally unclear. Consider the natural goal of the authority: make probabilities of being hired not depend on the serial number of a candidate and, therefore, be equal to each other. A combination of the results from Proposition 8 with the analysis in Section 5 suggests that the policy introduced in this Section may *increase* discrimination that appears in the unrestricted problem. For example, it happens if the sequence q_{iT} decreases in i. The ideal policy must take into account the intertemporal incentives of the manager and not naively put a simple restriction on the feasible set of strategies.

8 Conclusion

As documented in the economic literature, see, e.g., Bertheau et al. (2023), hiring is difficult for firms, and one of the reasons is that the firms face time constraints while hiring candidates. This means that firms do not learn the potential workers' productivities perfectly (since it will take too long time) but instead acquire noisy information about those. In this paper, we model the process of sequential search with costly but flexible learning in each stage.

The hiring firm observes several candidates who arrive sequentially and can design interviews for each candidate individually. We show that the optimal learning strategy has a simple feature – the later the candidate appears (the higher the serial number she has), the easier questions she will be facing. That is, the optimal interviews are decreasing in their difficulty in time. However, it does not mean that the workers should try to be interviewed in the end since the probability of being hired as a function of time of arrival is not necessarily increasing.

Our paper is the first step in studying sequential search with flexible and endogenous information acquisition. Therefore, many research questions are left for the future. For instance, we study only the situation in which the candidates are ex-ante identical, and the order of their arrival is random. The problem of studying a similar problem with a priori heterogeneity in workers' productivities and choice of order of the candidates is interesting and intriguing.

Another suggestion for future research is to consider a model similar to ours but with an opportunity for recall. We suspect that the decreasing difficulty property will remain present in this class of problems.

A Proofs

A.1 Intermediate results

Lemma A1. *Under Assumption 1, $p_V^L, p_V^H \in (0,1)$.*

Proof. Assume the contrary. Let Assumption 1 be satisfied. Let for a given V the employer chooses the optimal signal π with the pair of posterior beliefs p_V^L, p_V^H such that at least one of them belongs to the boundary $\{0,1\}$ and In this case, the inequality between slopes

$$1 - \lambda c'(p_V^H) > -\lambda c'(p_V^L)$$

holds. If $p_V^L = 0$ then by Assumption 1 inequalities

$$1 - \lambda c'(p_V^H) > -\lambda \lim_{p \to 0^+} c'(p) > 1 - \lambda c'(\mu)$$

hold and, therefore, because c is convex, inequality $\mu > p_V^H$ holds. If $p_V^H = 1$ then by Assumption 1 inequalities

$$-\lambda c'(p_V^L) < 1 - \lambda \lim_{p \to 1^-} c'(p) < -\lambda c'(\mu)$$

hold and, therefore, because c is convex, inequality $\mu < p_V^L$ holds. In both cases, the Bayesian consistency condition for π is violated. □

Lemma A2. *Both p_V^L and p_V^H are differentiable with respect to V in (V_L, V_H).*

Proof. Under Lemma 1, the concave closure of ϕ as defined in the static problem (2) for $p \in [p_V^L, p_V^H]$ is a straight line that is tangent to ϕ at p_V^L and p_V^H. This tangent is characterized by the following equality for $p \in [p_V^L, p_V^H]$

$$V - \lambda c(p_V^L) - \lambda c'(p_V^L)(p - p_V^L) = p_V^H - \lambda c(p_V^H) - [\lambda c'(p_V^H) - 1](p - p_V^H) \qquad \text{(A.1)}$$

such that

$$\lambda c'(p_V^L) = \lambda c'(p_V^H) - 1 \qquad \text{(A.2)}$$

By virtue of strict convexity of c and (A.2), we can implicitly define p_V^H as a continuously differentiable function of p_V^L. Using this in (A.1), we have that

$$V = p_V^H - \lambda c(p_V^h) - [\lambda c'(p_V^h) - 1](p - p_V^h) - [-\lambda c(p_V^L) - \lambda c'(p_V^L)(p - p_V^L] \qquad \text{(A.3)}$$

Using (A.2) yields:

$$0 = -V + p_V^H - \lambda c(p_V^H) + \lambda c(p_V^L) + \lambda c'(p_V^L)(p_V^H - p_V^L) \qquad \text{(A.4)}$$

Next, note that the RHS is a continuously differentiable function of p_V^L. Moreover its derivative with respect to p_V^L is given by:

$$(1 - \lambda c'(p_V^H)) \cdot (p_V^H)' + \lambda c'(p_V^L) + \lambda c'(p_V^L) \cdot (p_V^H)' - \lambda c'(p_V^L) + \lambda c''(p_V^L)(p_V^H - p_V^L) \\ = \lambda c''(p_V^L)(p_V^H - p_V^L) > 0$$

where the last equality comes from (A.2). The implicit function theorem implies that p_V^L is a continuously differentiable function of V and consequently so is p_V^H. □

Lemma A3. *Both p_V^L and p_V^H are increasing with respect to V in (V_L, V_H).*

Proof. Differentiating tangent optimality conditions (A.2),(A.4) with respect to V gives the system

$$\begin{cases} \frac{\partial p_V^L}{\partial V} = \frac{1}{\lambda c''(p_V^L)(p_V^H - p_V^L)} \\ \frac{\partial p_V^H}{\partial V} = \frac{1}{\lambda c''(p_V^H)(p_V^H - p_V^L)}, \end{cases} \tag{A.5}$$

By the convexity of c and because inequality $p_V^H > p_V^L$ holds, both derivative are positive. □

A.2 Proof of Lemma 2

For every $V \in (V_L, V_H)$, the optimal signal π_V assigns to the two respective posteriors, p_V^L and p_V^H, probability

$$\pi_V(p_V^L) = \frac{p_V^H - \mu}{p_V^H - p_V^L} \text{ and } \pi_V(p_V^H) = \frac{\mu - p_V^L}{p_V^H - p_V^L},$$

and the employer's indirect expected utility in (V_L, V_H) is

$$g(V) = \pi_V(p_V^H)\big(p_V^H - \lambda c(p_V^H)\big) + \pi_V(p_V^L)\big(V - \lambda c(p_V^L)\big).$$

Since p_V^L and p_V^H are differentiable in V, so is g. By the Envelope Theorem, we have

$$g'(V) = \pi_V(p_V^L) > 0.$$

Thus, g is strictly increasing in (V_L, V_H). Then, simple algebra yields

$$\frac{\partial \pi_V(p_V^L)}{\partial V} = \frac{\partial p_V^H}{\partial V}(\mu - p_V^L) + \frac{\partial p_V^L}{\partial V}(p_V^H - \mu),$$

which, by Lemma A3, is non-negative. Therefore, g is convex.

A.3 Proof of Lemma 3

Part 1 follows directly from (7) combined with Lemma 2.

By definition we have $V_T = 0$. Then, Part 2 follows directly from the fact that $g(V) \in (V_L, V_H)$ for all $V \in [0, V_H)$.

A.4 Proof of Theorem 1

The proof follows directly from from Lemmas 1 and 3.

In particular, by $V_T = 0$, we get $V_T < V_L$. Hence, $\text{supp}(\pi_{V_T}) = \{\mu\}$ and $\alpha_T(\mu) = 1$.

Moreover, for every $i \in \{1, \dots, T-1\}$, we have $V_L < V_i < V_H$, and therefore $\text{supp}(\pi_{V_i}) = \{p_{V_i}^L, p_{V_i}^H\}$ with $\alpha_i(p_{V_i}^L) = 0$ and $\alpha_i(p_{V_i}^H) = 1$.

A.5 Proof of Theorem 2

For the first part, it is sufficient to show that both optimal posterior beliefs in the static problem (6) or increasing functions of the outside option. It follows from the proof of Lemma 2.

From Proposition 1 and Lemma 1 continuation value V_1 converges to V_H when $T \to \infty$. In the solution to problem (6) with an outside option V_H, the solution to the first-order conditions from Lemma 1 implies that the lower optimal posterior equals the prior. Therefore, by the continuity $\lim_{T\to\infty} p^L_{V_1} = \mu$ holds and the high posterior belief converges to V_H.

A.6 Proof of Proposition 1

(i) $\iff$ (ii): For each $s \in \{H, L\}$, we have:

$$p_i^s = \frac{\mu}{\mu + (1-\mu)\frac{\sigma_i(s|B)}{\sigma_i(s|G)}} > p_i^s = \frac{\mu}{\mu + (1-\mu)\frac{\sigma_j(s|B)}{\sigma_j(s|G)}} \iff \frac{\sigma_i(s \mid G)}{\sigma_i(s \mid B)} > \frac{\sigma_j(s \mid G)}{\sigma_j(s \mid B)}$$

(i) $\Rightarrow$ (iii): By the Bayes rule the passing probability under test k equals to $(\mu - p_k^L)/(p_k^H - p_k^L)$. The required follows from the inequalities

$$\frac{\mu - p_j^L}{p_j^H - p_j^L} > \frac{\mu - p_j^L}{p_i^H - p_j^L} < \frac{\mu - p_i^L}{p_i^H - p_i^L}.$$

(iii) $\Rightarrow$ (i): Let the passing probability under test i is lower than under test j, but signal π_i is not more difficult than π_j. In this case at least one inequality $p_j^H \geq p_i^H, p_j^L \geq p_i^L$ holds. In the first case for a candidate $\mu = p_i^H$ and in the second case for a candidate $\mu = p_j^L$ the passing probability of test i is higher. Thus, the signal π_i has to be more difficult than π_j.

A.7 Proof of Proposition 2

First, he symmetry of c implies that $c'(z) = 0$. Boundary assumption 1 garanties existence of the point $p' \in (z, 1)$ such that equality $c'(p') = \frac{1}{2\lambda}$ holds. The pair of points $(p_\mu^L, p_\mu^H) = (p', p'')$, where $p'' = 2z - p'$ is the solution to the optimality conditions (A.2, A.4) for the optimal posterior beliefs with an outside option $V = z$. If $T = 2$ then $V = \mu$ holds and, therefore, if $\mu = z$ holds then a pair (p', p'') is a pair of optimal posterior beliefs. Rearranging the optimality condition A.4) gives

$$\frac{p_\mu^H - \mu}{p_\mu^H - p_\mu^L} = \frac{1}{2},$$

thus $q_{12} = q_{22}$.

Second, we calculate the derivative of q_{22} with respect to the outside option, that in the case $T = 2$ equals to μ

$$\frac{dq_{22}}{d\mu} = \left(\frac{p_\mu^H - \mu}{p_\mu^H - p_\mu^L}\right)'_\mu = \frac{((p_\mu^H)' - 1)(p_\mu^H - p_\mu^L) - (p_\mu^H - \mu)((p_\mu^H)' - (p_\mu^L)')}{(p^H - p^L)^2}$$

Optimal posteriors p_μ^H and p_μ^L are symmetric around z, therefore, equality $(p_\mu^H)' = (p_\mu^L)'$ holds, and the sign of the derivative is determined by the fact whether $(p_\mu^H)'$ is larger than 1. We substitute the expression for $(p_\mu^H)'$ using conditions from Lemma A.5:

$$(p_\mu^H)' - 1 = \frac{1}{\lambda c''(x^H)(p_\mu^H - p_\mu^L)} - 1 = \frac{c'(p_\mu^H) - c'(p_\mu^L)}{c''(p_\mu^H)(p_\mu^H - p_\mu^L)} - 1 =$$
$$= \frac{1}{c''(p_\mu^H)(p_\mu^H - p_\mu^L)}\Bigg(c'(p_\mu^H) - c'(z) - c''(p_\mu^H)(p_\mu^H - z) + \tag{A.6}$$
$$+ \Big(c'(z) - c'(p_\mu^L) - c''(p_\mu^L)(z - p_\mu^L)\Big)\Bigg)$$

where we substitute optimality conditions for λ and use equality $c''(p_\mu^L) = c''(p_\mu^H)$.

Once we assume the behavior of $c(p)$ on $(z, 1)$, the symmetry assumption of function $c(p)$ pins down the behavior on of $c(p)$ on $(0, z)$. In particular, of $c(p)$ is linear on $(z, 1)$ then $c(p)$ is linear on $(0, z)$; if $c(p)$ is convex on $(z, 1)$ then $c(p)$ is concave on $(0, z)$; if $c(p)$ is concave on $(z, 1)$ then $c(p)$ is convex on $(0, z)$. Therefore, if $c(p)$ is linear on $(z, 1)$, then both terms in the brackets expression (A.6) equal to 0 and $q_{12} = q_{22}$. If $c(p)$ is convex on $(z, 1)$, then by the intermediate value theorem both terms in the brackets expression (A.6) are negative and $q_{12} > q_{22}$ if $\mu \approx z + \varepsilon$. If $c(p)$ is concave on $(z, 1)$, then by the intermediate value theorem both terms in the brackets expression (A.6) are positive and $q_{12} > q_{22}$ if $\mu \approx z + \varepsilon$.

A.8 Proof of Proposition 3

If $T = 2$ then the Bayes rule implies that the first candidate is favored if $p_\mu^H + p_\mu^H < 0.5$. If c is negative entropy, then the low and high posteriors are given by:

$$p_\mu^L = \frac{e^{\frac{\mu}{\lambda}} - 1}{e^{\frac{1}{\lambda}} - 1} \qquad\qquad p_\mu^H = \frac{e^{\frac{1}{\lambda}} - e^{\frac{1-\mu}{\lambda}}}{e^{\frac{1}{\lambda}} - 1}$$

Note that the function $p_\mu^H + p_\mu^L$ is strictly increasing in μ. Moreover, it is concave for $\mu \in \left(0, \frac{1}{2}\right)$ and convex on $\mu \in \left(\frac{1}{2}, 1\right)$. Finally, it intersects the function 2μ at $\left\{0, \frac{1}{2}, 1\right\}$. Consequently, it lies strictly above 2μ for $\mu \in \left(0, \frac{1}{2}\right)$ and strictly below 2μ for $\mu \in \left(\frac{1}{2}, 1\right)$.

A.9 Proof of Proposition 4

Simple algebra shows that the system has unique solution $p_V^L = V - \frac{1}{4\lambda}, p_V^H = V + \frac{1}{4\lambda}$. Substituting the solution gives the value of the problem as $g(V) = V + \lambda(\mu - V + \frac{1}{4\lambda})^2$.

We show that for a given $T > 2$ inequality $q_{1T} > q_{2T}$ holds. Using the derived expressions above we get that

$$q_{1T} = 2\lambda\Big(\mu - V_1 + \frac{1}{4\lambda}\Big); \qquad q_{2T} = 4\lambda^2\Big(V_1 - \mu + \frac{1}{4\lambda}\Big)\Big(\mu - V_2 + \frac{1}{4\lambda}\Big),$$

moreover equality

$$V_1 = V_2 + \lambda\Big(\mu - V_2 + \frac{1}{4\lambda}\Big)^2$$

holds. We denote $t = \mu - V_2 + \frac{1}{4\lambda}$, thus,

$$q_{1T} = 2\lambda(t - \lambda t^2)$$
$$q_{2T} = 4\lambda^2 t\Big(\frac{1}{2\lambda} - t + \lambda t^2\Big)$$

Therefore, the inequality $q_{1T} > q_{2T}$ is equivalent to the inequality $\frac{1}{2\lambda} > t$. Because inequality $V_2 > \mu - \frac{1}{4\lambda}$ hold for all $T > 2$, inequality $q_{1T} > q_{2T}$ also holds.

By the Bayes rule the inequality $q_{tT} > q_{t+1T}$ is equivalent to the inequality $q_{1T-t+1} > q_{2T-t+1}$, therefore, the inequality holds.

Finally, if $T = 2$ then $q_{12} = 2\lambda(\mu - V_1 + \frac{1}{4\lambda}) = \frac{1}{2}$ because in this case $V_1 = \mu$. Therefore, $q_{T-1T} = q_{TT}$ for all T.

A.10 Proof of Proposition 5

For convenience we denote $x = \frac{1}{\lambda}$. Matějka and McKay (2015) analysis imply that the unconditional probabilities of choosing the first candidate in the case of $T = 2$ and $T = 3$ equal

$$q_{12} = -e^{\mu x}\frac{-1 + e^{\mu x} + \mu - \mu e^x}{(e^x - e^{\mu x})(-1 + e^{\mu x})}; \quad q_{13} = -e^{g(\mu)x}\frac{-1 + e^{g(\mu)x} + \mu - \mu e^x}{(e^x - e^{g(\mu)x})(-1 + e^{g(\mu)x})}$$

We consider the behavior of q_{12} and q_{13} as a function x and, therefore, also consider $g(\mu)$ as a function of x and use the notation $q_{12}(x), q_{13}(x)$ and $R(x)$ correspondingly.

The proof consists of three parts. We first show that $\lim\limits_{x\to 0} q_{12}(x) = \frac{1}{2}$ holds. Later, we consider the Taylor approximation of the function $R(x)$ at $x = 0$. We show that $R(x) = \mu + (\frac{1}{8}\mu - \frac{1}{8}\mu^2)x + o(x)$ holds. Finally, we show that if $R(x)$ admits such form then $\lim\limits_{x\to 0} q_{13}(x) = \frac{3}{8}$ holds.

Part 1.

We use the second-order Taylor expansion as

$$e^x = 1 + x + \frac{p^2}{2} + o(p^2), \quad e^{\mu x} = 1 + \mu x + \frac{(\mu x)^2}{2} + o(p^2)$$

Substituting these expressions to the formula for q_{12} gives

$$q_{12}(x) = -e^{\mu x}\frac{\frac{1}{2}p^2(\mu^2 - \mu) + o(p^2)}{p^2\mu(1 - \mu) + o(p^2)}.$$

Clearly, $\lim\limits_{x\to 0} -e^{\mu x} = -1$ holds. Therefore,

$$\lim_{x\to 0} q_{12}(x) = -\frac{\frac{1}{2}p^2(\mu^2 - \mu) + o(p^2)}{p^2(\mu - \mu^2) + o(p^2)} = \frac{1}{2}.$$

Part 2.

Matějka and McKay (2015) analysis imply that value $R(x)$ can be expressed as

$$R(x) = \frac{1}{x}\left(\mu \log\left(q_{12}(x)e^{x}+(1-q_{12}(x))e^{\mu x}\right)+(1-\mu)\log\left(q_{12}(x)+(1-q_{12}(x))e^{\mu x}\right)\right).$$

Clearly, $q_{13}(x)$ is analytic function on $x \in (0, 0+\varepsilon)$, therefore, $R(x)$ is also analytic on $x \in (0, 0+\varepsilon)$. We derive the first-order Taylor expansion for function $R(x)$. For that we use the second-order Taylor expansion of $q_{12}(x)$

$$q_{12}(x) = \frac{1}{2} + \beta x + \gamma p^2 + o(p^2),$$

where β, γ are some real numbers. Plugging in the expressions for exponential function and $q_{12}(x)$ inside the first log results into expression

$$\log\left(1 + x(\frac{1}{2} + \frac{1}{2}\mu) + p^2(\frac{1}{4} + \beta + \frac{\mu^2}{4} - \beta\mu) + o(p^2)\right)$$

Using the second-order Taylor expansion for log(.) and multiplying the resulting expression by μ leads to the following expression for the first term in the sum for $R(x)$:

$$x\frac{1}{2}(1+\mu)\mu + p^2\mu(\frac{1}{4} + \beta + \frac{\mu^2}{4} - \beta\mu - \frac{1}{2}(\frac{1}{2} + \frac{1}{2}\mu)^2) + o(p^2).$$

Similarly, we get the expression for the second term in the sum for $R(x)$:

$$x\frac{1}{2}\mu(1-\mu) + (1-\mu)(\frac{\mu^2}{8} - \beta\mu) + o(p^2).$$

Plugging them into the formula for $R(x)$, we obtain

$$R(x) = \frac{1}{x}\left(x\mu + \frac{1}{8}\mu(1-\mu)p^2 + o(p^2)\right) = \mu + \frac{1}{8}(\mu - \mu^2)x + o(x).$$

Part 3.

For convenience we denote $\alpha = \frac{1}{8}(\mu - \mu^2)$. Therefore, we write $R(x)x = \mu x + \alpha p^2 + o(p^2)$ and the second-order Taylor expansion for $e^{R(x)x}$ becomes

$$e^{R(x)x} = 1 + \mu x + p^2(\alpha + \frac{\mu^2}{2}) + o(p^2).$$

We substitute this expression into the fraction for the formula for $q_{13}(x)$:

$$q_{13}(x) = -e^{R(x)x}\frac{p^2(\alpha + \frac{1}{2}(\mu^2 - \mu)) + o(p^2)}{p^2(\mu - \mu^2) + o(p^2)}$$

Clearly, $\lim_{x\to 0} -e^{R(x)x} = -1$ holds. Therefore,

$$\lim_{x\to 0} q_{13}(x) = -\frac{p^2(\alpha + \frac{1}{2}(\mu^2 - \mu)) + o(p^2)}{p^2(\mu - \mu^2) + o(p^2)} = \frac{1}{2} - \frac{\alpha}{\mu - \mu^2}.$$

From the Part 2 we get that $\alpha = \frac{1}{8}(\mu - \mu^2)$. Therefore, $\lim_{x\to 0} q_{13}(x) = \frac{3}{8}$. By the Part 1 $\lim_{x\to 0} q_{12}(x) = \frac{1}{2}$, thus, $\lim_{x\to 0} q_{23}(x) = \lim_{x\to 0} q_{33}(x) = \frac{5}{16}$.

A.11 Proof of Proposition 6

We analyze the problem as a collection of static problems (2) at period i with an outside option V_i. We compare the solution of the problem to the solution when $\mu_i = \mu$ for all i.

Recall that for any i, the value V_i does not depend on the history. Therefore, for $i > i'$, problem (2) does not change, and DM chooses the same optimal interview structure.

In the problem at the period $i = i'$, the outside option does not change, but the prior belief changes. Optimal posteriors are independent of the prior belief; therefore, DM does not change the difficulty of the optimal interview.

In the problem at period $i = i' - 1$, the continuation value V_i becomes larger. It leads to two observations: by Lemma 1 DM chooses interview that is more difficult, and the continuation value V_{i-1} increases. Applying such an argument iteratively for all interviews before $i' - 1$, we get that the employer chooses more difficult interviews for all $i < i'$.

A.12 Proof of Proposition 7

Similarly to the static problem, we denote $g(\mu_1, \mu_2)$ as the value function of the problem with two candidates, where μ_1 is the expected quality of the first agent and μ_2 is the expected quality of the second agent. To derive the optimal order, we apply the Taylor expansion argument. Function $g(\mu_1, \mu_2)$ is, clearly, analytical. Therefore, the inequality between $g(\mu + \varepsilon, \mu)$ and $g(\mu, \mu + \varepsilon)$ is equivalent to the inequality between $g_{\mu_1}(\mu, \mu)$ and $g_{\mu_2}(\mu, \mu)$, where the subscript refers to the partial derivative.

Using definition of the function $g(\mu_1, \mu_2)$ we can write

$$g(\mu_1, \mu_2) = \frac{p^H_{\mu_2} - \mu_1}{p^H_{\mu_2} - p^L_{\mu_2}}(\mu_2 - c(p^L_{\mu_2})) + \frac{\mu_{\mu_2} - p^L_{\mu_2}}{p^H_{\mu_2} - p^L_{\mu_2}}(p^H_{\mu_2} - c(p^H_1))$$

for any μ_1, μ_2, if DM acquires the information. Therefore, in the optimum we can express $g_{\mu_1}(\mu_1, \mu_2)$ and $g_{\mu_2}(\mu_1, \mu_2)$ as

$$g_{\mu_1}(\mu_1, \mu_2) = \frac{1}{p^H_{\mu_2} - p^L_{\mu_2}}\Big(- \mu_2 + c(p^L_{\mu_2}) + p^H_{\mu_2} - c(p^H_{\mu_2})\Big);$$

$$g_{\mu_2}(\mu_1, \mu_2) = \frac{p^H_{\mu_2} - \mu_{\mu_2}}{p^H_{\mu_2} - p^L_{\mu_2}},$$

where the first equality comes from the fact that optimal posteriors are independent of the prior, and the second comes from the Envelope theorem. Thus, inequality between partial derivatives $g_{\mu_1}(\mu, \mu)$ and $g_{\mu_2}(\mu, \mu)$ is equivalent to the inequality between the $c(p^L_{\mu_2})$ and $c(p^H_{\mu_2})$. We get the desired result about the optimal order by the proof of the Proposition 2.

B Reformulation of the static problem

Lemma B4. *There exists a bijection between static problems (2), in which DM chooses posterior distribution and static problems*

$$\max_{q\in[0,1]} U(q,V), \tag{B.1}$$

in which DM chooses marginal probability. For any solution (p^L, p^H) to problem (2) exists unique solution q^* to problem (B.1) and vice versa, and equality $g(V) = U(q^*, V)$ holds.

Additionally, problem (B.1) is concave in q, with unique solution, and the interior solution of (B.1) is decreasing in V.

Proof. Instead of relying on the concavification technique to solve the static problem (2), we introduce the unconditional probability explicitly as a choice variable. In the concavification technique, the optimal unconditional probability is derived as a function of the two posterior beliefs that satisfy Bayesian consistency. Instead, we show the opposite: that optimal posteriors can be derived from the unconditional probability of choice.

We first analyze the case in which inequality $p^H > \mu > p^L$ holds. That is, when $V \in (V_L, V_H)$. Using the fact that support of the optimal posterior distribution has no more than two points we rewrite the static problem (2) as

$$\begin{aligned} \max_{(q,p^H,p^L)\in[0,1]^3} & \{q(p^H - \lambda c(p^H)) + (1-q)(V - \lambda c(p^L))\} \\ \text{s.t.} \quad & qp^H + (1-q)p^L = \mu, \\ & p^H \geq p^L. \end{aligned} \tag{B.2}$$

We denote the objective in the above problem as $\tilde{U}(q, p^H, p^L)$. Using the identity from the multivariable calculus we can write $\max_{q,p^H,p^L} \tilde{U}(q,p^H,p^L) = \max_q \max_{p^H,p^L} \tilde{U}(q,p^H,p^L)$. Therefore, problem (B.2) can always be solved sequentially finding optimal p^L, p^H given q and then optimize over q. To show the equivalence between problems, we need to show first that for given q, there is only one pair of optimal (p^L, p^H) and second that there is a unique optimal q.

To show that exists a unique pair of optimal (p^L, p^H) given q we observe that the interior solution to the static problem (2) should satisfy necessary optimality conditions in problem (B.2). In particular, for given q optimal (p^L, p^H) should satisfy the system

$$\begin{cases} -\lambda c'(p^L) = 1 - \lambda c'(p^H) \\ qp^H + (1-q)p^L = \mu. \end{cases}$$

We show that given $q \in (0,1)$ the system has a unique solution (p^L, p^H). We rewrite the first equation as $p^L = (c')^{-1}(c'(p^H) - \frac{1}{\lambda})$. This expression define a function $p^L(p^H)$. Indeed, $c'(p^H)$ is increasing in p^H, therefore mapping $p^L(p^H)$ is also increasing and the mapping defines unique p^L for any p^H.

We rewrite the second equation as $q = (\mu - p^L)/(p^H - p^L)$. Simple algebra shows that the derivative of the right-hand side with respect to p^H is positive and, therefore, the right-hand side is increasing in p^H. Therefore, for any given q exists unique p^H and, thus, for any given q exists unique pair (p^L, p^H) that solves the system above. Further, we write $p^L(q), p^H(q)$ to emphasize the dependence of optimal posteriors from the marginal distribution.

Following Fosgerau et al. (2023) we show that problem (B.1) is concave. Without abuse of notation, we omit V in the argument and denote the objective function as U. The derivative equals to

$$U'(q) = -p^H(q) + \lambda c(p^H(q)) + (V - \lambda c(p^L(q)))+ \\ +(1-q)((p^H(q))' - \lambda c'(p^H(q))(p^H(q))') - (q)\lambda c'(p^L(q))(p^L(q))'$$

where $(p^H(q))', (p^L(q))'$ are the derivative of the posteriors with respect to q. Using optimality condition $\lambda c'(p^H) - \lambda c'(p^L) = 1$ and differentiating Bayesian consistency to get $(1-q)(p^H(q))' + q(p^L(q))' = p^L(q) - p^H(q)$ we obtain

$$U'(q) = p^H(q) - \lambda c(p^H(q)) - (V - \lambda c(p^L(q))) + \lambda c'(p^L(q))(p^H(q) - p^L(q)).$$

Differentiating the expression with respect to q one more time and using optimality condition for posteriors result in

$$U''(q) = \lambda c''(p^L(q))(p^H(q) - p^L(q))(p^L(q))'.$$

We show that inequality $(p^L(q))' < 0$ holds. Combining optimality condition for posteriors and differentiable Bayesian consistency condition we obtain that equality $(p^L(q))'\Big(q\frac{c''(p^L(q))}{c''(p^H(q))} + 1 - q\Big) = p^L(q) - p^H(q)$ holds. Thus $(p^L(q))' < 0$ holds and $U''(q) < 0$ holds and problem (B.1) is concave. Therefore, problem (B.1) has a unique solution that is determined from the first-order condition or on the boundary. However, because the inequality $p^H > \mu > p^L$ holds, the optimal q is interior and uniquely determined from the first-order condition.

If $V \notin (V_L, V_H)$ then DM chooses degenerate distribution, optimal q equals to 0 if $V \le V_L$ and equals to 1 if $V \ge V_H$.

To get comparative statics of the optimal q with respect to V we employ standard supermodularity argument: optimal interior q is decreasing in V, because the mixed derivative of $\frac{\partial^2 U(q,V)}{\partial q \partial V} = -1$ is negative.

□

Lemma B4 allows to analyze a static problem with only one control variable. Optimal posteriors could be obtained from the optimality conditions, because there is a one-to-one mapping between optimal marginal probability and optimal posteriors.

We compare the solution to the unrestricted and restricted problems. Using Lemma B4 without abuse of notation, we rewrite the unrestricted dynamic problem as

$$\max_{r_i \in [0,1]} U(r_i, V_i) \quad \forall\, i \neq T, \\ \text{s.t.} \\ V_i = \Big\{ \max_{r_{i+1} \in [0,1]} U(r_{i+1}, V_{i+1}) \Big\}, \\ V_T = 0, \tag{B.3}$$

where we use r as a decision variable to avoid possible confusion between the probability of choosing a candidate i in the dynamic problem and the probability of candidate i passing a test. Similarly, we rewrite the restricted problem as

$$\max_{r \in [0,1]} U(r, V_1(r, \mu)), \tag{B.4}$$

where $V_1(r, \mu) = U(r, \underbrace{U(r, \dots, U(r, \mu))}_{T-3})$. We distinguish the restricted problem from the unrestricted one, including r in the second argument of the objective function.

B.1 Proof of Proposition 8

We assume that the solution to the restricted problem is interior and later show that it is indeed optimal. Optimal interior q^* solves

$$\frac{\partial U}{\partial r}(q^*, V_1(q^*, \mu)) + \frac{\partial U}{\partial V}(q^*, V_1(q^*, \mu)) \times \frac{dU}{dr}(q^*, V_2(q^*, \mu)) = 0.$$

Using the chain rule we can express this first-order condition as

$$\frac{\partial U}{\partial r}(q^*, V_1(q^*, \mu)) + \sum_{j=2}^{T-1} \left(\frac{\partial U}{\partial r}(q^*, V_j(q^*, \mu)) \prod_{k=1}^{j-1} \frac{\partial U}{\partial V}(q^*, V_k(q^*, \mu)) \right) = 0. \tag{B.5}$$

We will show that the first term in the first-order condition is negative, that inequality $\frac{\partial U}{\partial r}(q^*, V_1(q^*, \mu)) < 0$ holds. We show this fact by the contradiction, analyzing the second term in the first-order condition.

First, we observe, that for general values r, V the following partial derivatives equal to

$$\frac{\partial U}{\partial V}(r, V) = 1 - r, \quad \frac{\partial^2 U}{\partial V \partial r}(r, V) = -1,$$

therefore, partial derivative $\frac{\partial U}{\partial V}(r, V)$ is always positive and function $\frac{\partial U}{\partial r}(r, V)$ is decreasing in V.

Second, let inequality $\frac{\partial U}{\partial r}(q^*, V_1(q^*, \mu)) \geq 0$ hold. The partial derivative of function in r is decreasing in the second argument, therefore, inequality $\frac{\partial U}{\partial r}(q^*, V_{i+1}(q^*, \mu)) > 0$ hold for all $i > 1$. Thus, the second term in the first-order condition (B.5) is positive. Therefore, the left-hand side of the equation (B.5) is positive. We reach a contradiction, thus, inequality $\frac{\partial U}{\partial r}(q^*, V_1(q^*, \mu)) < 0$ holds.

To compare q^* with q^{**} we observe that inequality $\frac{\partial U}{\partial r}(q^*, \mu) > 0$ holds. Indeed, if this inequality does not hold, then the argument from the above paragraph suggests that the left-hand side of the first-order condition (B.5) is negative. Optimal q^{**} in the $T = 2$ case solves $\frac{\partial U}{\partial r}(q^{**}, \mu) = 0$. Function $\frac{\partial U}{\partial r}(r, V)$ is decreasing in r, therefore, inequality $q^{**} > q^*$ holds.

To compare q^* with q^{***} we consider the auxiliary static problem with an outside option $V_1(q^*, \mu)$:

$$\max_r \{U(r, V_1(q^*, \mu))\}$$

In this problem, the value of the outside option equals the obtained outside option value in the restricted problem. Let q^{****} be a solution to this problem, therefore

q^{****} solves $\frac{\partial U}{\partial r}(q^{****}, V_1(q^*, \mu)) = 0$. Function $\frac{\partial U}{\partial r}(r, V)$ is decreasing in r, therefore the inequality $q^* > q^{****}$ holds.

The unrestricted problem is a static problem with an outside option V_1. Clearly, the inequality $V_1 > V_1(q^*, \mu)$ holds. Optimal r in the static problem decreases in the outside option value V, therefore, inequality $q^{****} > q^{***}$ holds and by transitivity inequality $q^* > q^{***}$ also holds.

We show that the optimal q^* is interior by taking derivatives of the function $U(r, V_1(r, \mu))$ on the boundary. We consider the case $r = 0$, and the analysis of case $r = 1$ is identical. We observe that $V_i(0, \mu) = \mu$ for all $i < T$. Additionally, equality $\frac{\partial U}{\partial V}(0, V_i(0, \mu)) = 1$ holds. Therefore, using the left-hand side of the expression (B.5) we obtain that

$$\frac{\partial U}{\partial r}(r, V_1(r, \mu))\bigg|_{r=0+0} = (T-1)\frac{\partial U}{\partial r}(0, \mu) > 0.$$

The last inequality holds because the static problem with an outside option μ has an interior solution and by Lemma B4 function $U(r, \mu)$ is concave in r.

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