Reasonable doubt revisited

Elias Tsakas

Maastricht University

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Roadmap

Roadmap

Threshold choice rules

- Probability threshold (choice) rules are used in several disciplines:
  - Law (e.g., Kaplan, 1968; Tribe, 1971; Kaplow, 2012)
  - Medicine (e.g., Pauker & Kassirer, 1975, 1980)
  - Economics (e.g., Shavell, 1985; Andreoni, 1991; Kaplow, 2011)
  - Statistics (e.g., Neyman & Pearson, 1933)
  - Finance (e.g., Roy, 1952; Telser, 1955-56)
- **Our main question:** which threshold?
- We focus on the context of law, because:
  - Most of the discussion has taken place within the law literature.
  - In law the use of probability thresholds is normatively postulated.

Standard of reasonable doubt

- The standard of reasonable doubt is a (high) probability threshold such that *the juror prefers to convict the defendant iff the probability of guilt is above this threshold.*
- It is a threshold rule that induces a rational choice for every belief.
- *If it exists*, it constitutes the answer to our previous question.
- **Question becomes:** does the standard of reasonable doubt exist?

Main (Impossibility) Theorem

Generically, the standard of reasonable doubt exists if and only if the juror reasons only about the defendant's guilt/innocence.

Following our impossibility result

- When the use of a threshold rule is willingly chosen by the decision maker (e.g., in medicine or finance), more complicated strategies should be used if we wish to maintain rationality.
- When the use of a threshold rule is exogenously postulated (e.g., in law), some irrationalities must be accepted.
    - Selection of a threshold depends on attitude towards irrationalities: irrational convictions vs. irrational acquittals.
    - Implications for law.

Roadmap

Juror's frame

- Two agents, a (female) juror and a (male) defendant.
- A state space, Ω.
- Two basic complementary events, guilt (G) and innocence (I).
- The juror's frame, modelled by an algebra ($\mathcal{R}$).
  - The juror is either unaware or consciously disregards events outside $\mathcal{R}$.
  - The frame at the time of her decision: we do not model the process.
  - The juror always reasons about guilt/innocence ($G \in \mathcal{R}$ and $I \in \mathcal{R}$).
  - Reasoning only about guilt/innocence: $\mathcal{R} = \mathcal{G} := \{\Omega, G, I, \emptyset\}$.

Preferences

- Set of possible verdicts (alternatives) $X \subseteq [0, \infty]$.
  - Acquittal ($0 \in X$) and convictions ($X_+ := X \setminus \{0\}$).
- Preferences represented by the SDEU function $\mathbb{E}_\pi U_x = \int_\Omega U_x d\pi$.
  - The utility index $U_x : \Omega \to \mathbb{R}$ is $\mathcal{R}$-measurable.
  - The beliefs $\pi \in \Delta(\Omega, \mathcal{R})$ assign probabilities only to events in $\mathcal{R}$.
- Axiomatizations of SDEU need additional structure to uniquely identify beliefs (e.g., Fishburn, 1973; Karni et al., 1983; Karni, 1993).
- We impose less structure than frame-dependent EU (e.g., Ahn & Ergin, 2010; Karni & Vierø, 2013; Schipper, 2013).
- Assumption: $V_x := U_x - U_0$ for $x \in X_+$, where:
  - $V_x(\omega) < 0$ for all $\omega \in I$ (preference for acquitting the innocent).
  - $V_x(\omega) > 0$ for some $\omega \in G$ (nontriviality).

Choice and Rationality

- Decision problem: $\Gamma \subseteq X$ with $0 \in \Gamma$ (focus on binary $\Gamma = \{0, x\}$).
- Choice rule: $\sigma : \Delta(\Omega, \mathcal{R}) \to \Gamma$.
- (Probability) threshold (choice) rule: $\boxed{\sigma_p(\pi) = x \Leftrightarrow \pi \in D_p}$, with

$$D_p = \{\pi \in \Delta(\Omega, \mathcal{R}) : \pi(G) \geq p\}$$

- Rational choice rule: $\boxed{\sigma(\pi) = x \Leftrightarrow \pi \in C_x}$, with

$$C_x = \{\pi \in \Delta(\Omega, \mathcal{R}) : \mathbb{E}_\pi V_x \geq 0\}.$$

Standard of reasonable doubt

Definition

$p_x \in [0,1]$ is the standard of reasonable doubt for $x \in X_+$ if $C_x = D_{p_x}$.

- The standard of reasonable doubt induces a rational threshold rule.
- The juror prefers to convict the defendant ($\mathbb{E}_\pi V_x \geq 0$) iff the probability she attaches to guilt is above the threshold ($\pi(G) \geq p_x$).
- This definition is common in the literature:
  - Foundations of reasonable doubt (e.g., Kaplan, 1968; Tribe, 1971; Andreoni, 1991).
  - Applications and examples within elsewhere-focused papers (e.g., Feddersen & Pesendorfer, 1998; Kamenica & Gentzkow, 2011).

Roadmap

Existence of standard of reasonable doubt

Main (Impossibility) Theorem

The standard of reasonable doubt p_x exists iff V_x is $\mathcal{G}$-measurable.

- $\boxed{\text{When is } V_x \text{ actually } \mathcal{G}\text{-measurable?}}$
  - The juror reasons only about events in $\mathcal{G}$ (generic).
  - The juror reasons about events outside $\mathcal{G}$ (circumstances), but she finds them irrelevant for her decision (nongeneric).

  Identification is not possible (Schipper, 2013).
- $\boxed{\text{What if we still use a threshold rule?}}$
  - We have to accept some irrationalities (we come back to this).

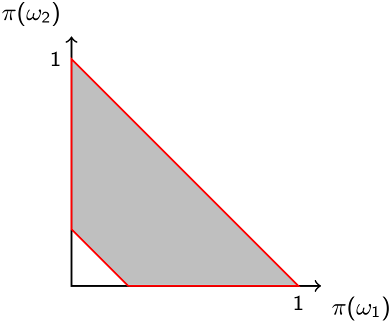
Example

- $\Omega = \{\omega_1, \omega_2, \omega_3\}$, $G = \{\omega_1, \omega_2\}$, $\mathcal{R} = 2^\Omega$.

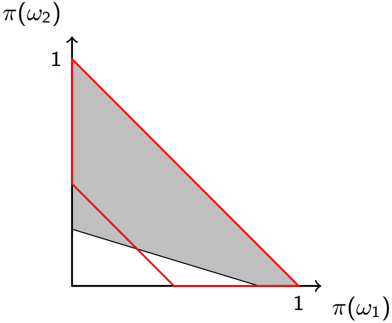
$$\begin{aligned} U_x(\omega_1) &= -2x^2 + 10x &\Rightarrow\quad & V_x(\omega_1) = -2x^2 + 10x \text{ (unintentional guilt)} \\ U_x(\omega_2) &= 10x^2 - 2x &\Rightarrow\quad & V_x(\omega_2) = 10x^2 - 2x \text{ (intentional guilt)} \\ U_x(\omega_3) &= -x &\Rightarrow\quad & V_x(\omega_3) = -x \text{ (innocence).} \end{aligned}$$

1. $\Gamma = \{0, 1\}$: she reasons but does not care about his intentions.
   - $V_1(\omega_1) = 8$ and $V_1(\omega_2) = 8$.
   - V_1 is $\mathcal{G}$-measurable.
   - $C_1 = D_{1/9}$ ($p_1 = 1/9$ is the standard of reasonable doubt for $x = 1$).
2. $\Gamma = \{0, 2\}$: she reasons and cares about his intentions.
   - $V_2(\omega_1) = 16$ and $V_2(\omega_2) = 36$.
   - V_2 is not $\mathcal{G}$-measurable.
   - $C_2 \neq D_p$ for all $p \in [0, 1]$ (there is no standard of reasonable doubt for $x = 2$).

Graphical illustration/Sketch of the proof



(a) V_1 is $\mathcal{G}$-measurable

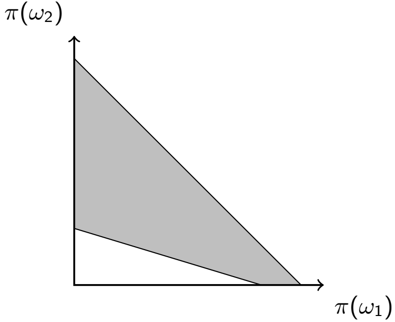


(b) V_2 is not $\mathcal{G}$-measurable

Roadmap

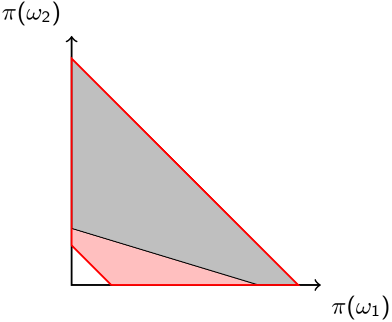
Irrationalities

- If p_x does not exist, every threshold rule induces irrationalities:



Irrationalities

- If p_x does not exist, every threshold rule induces irrationalities:
  - Irrational convictions (false negatives: $N_x^p := D_p \setminus C_x$)



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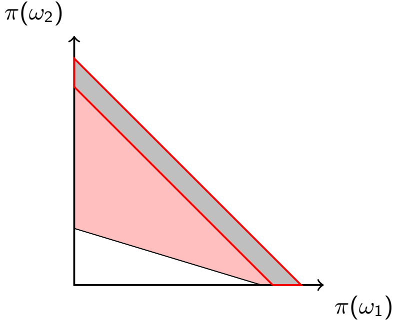

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- If p_x does not exist, every threshold rule induces irrationalities:
  - Irrational convictions (false negatives: $N_x^p := D_p \setminus C_x$)
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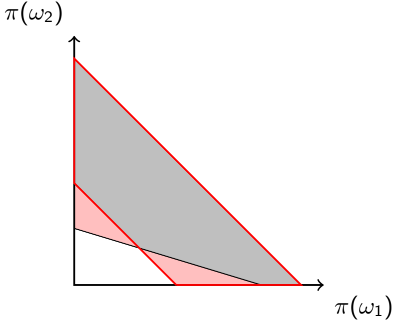
  - Both (false negatives and false positives)



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  - Irrational convictions (false negatives: $N_x^p := D_p \setminus C_x$)
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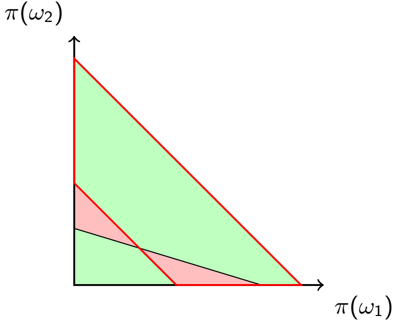
  - Both (false negatives and false positives)
- The rational verdicts are denoted by $R_x^p := \Delta(\Omega, \mathcal{R}) \setminus (N_x^p \cup P_x^p)$.



Aversion to irrationalities

- If we must pick a threshold rule, which one shall we choose?
  - It depends on attitude for irrational convictions vs. irrational acquittals.
- Such preferences are not formally introduced.
- We assume aversion to irrationalities (in general):
  - p dominates p' (p yields "fewer" irrationalities than p') iff $R_x^p \supsetneq R_x^{p'}$
- We cannot say which threshold, but we can say which thresholds not.

Weak standards of reasonable doubt

Definition

$p_x^w \in [0,1]$ is a weak standard of reasonable doubt for $x \in X_+$, if

$$\max\{0, p_x^\ell\} \geq p_x^w \geq \min\{p_x^u, 1\},$$

where

$$p_x^u := \min\{p \in [0,1] : C_x \supseteq D_p\}$$

is the upper (weak) standard of reasonable doubt and

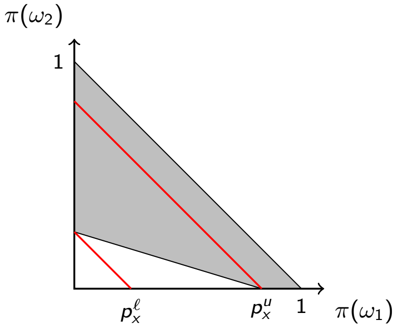
$$p_x^\ell := \max\{p \in [0,1] : C_x \subseteq D_p\}$$

is the lower (weak) standard of reasonable doubt.

Theorem

p is a weak standard of reasonable doubt iff it is not dominated.

Graphical illustration/Sketch of the proof



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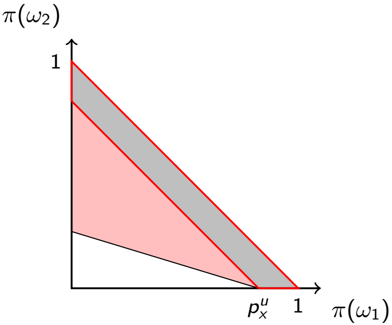

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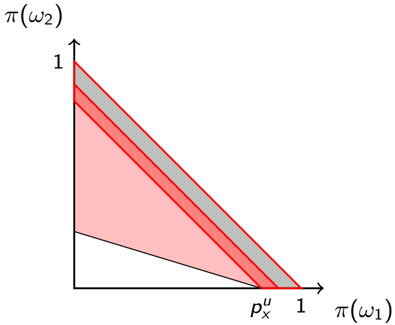

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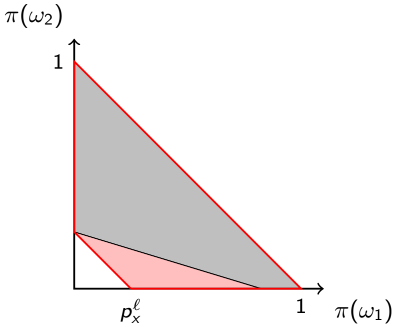

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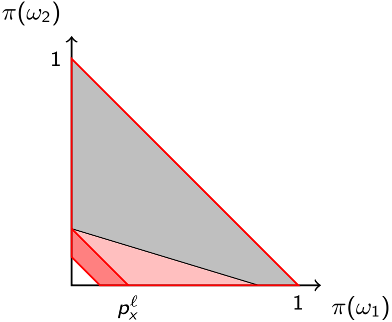

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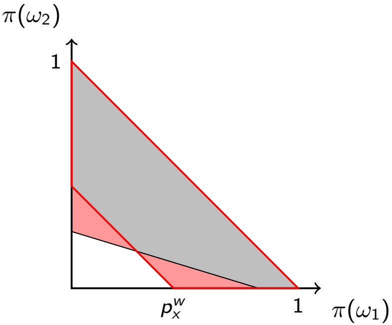

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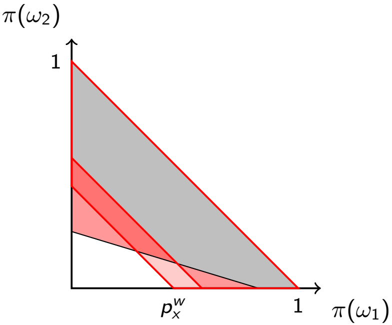

Intrerpretation of the weak standards

- Upper standard: extreme aversion to irrational convictions.
- Lower standard: extreme aversion to irrational acquittals.
- Additional structure is needed to select a threshold from $[p_x^\ell, p_x^u]$.
  - Answer to the debate between Kaplan (1968) and Tribe (1971).
- Similar idea in medicine (Pauker & Kassirer, 1980).

Existence of weak standards

Proposition

The standard of reasonable doubt p_x *exists iff* $p_x^u = p_x^\ell$.

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p_x^ℓ *always exists.*

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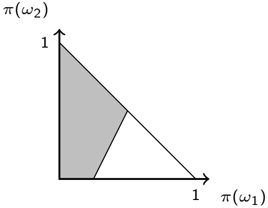

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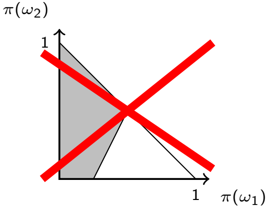

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The standard of reasonable doubt p_x *exists iff* $p_x^u = p_x^\ell$.

Proposition

p_x^ℓ *always exists.*

Proposition

p_x^u *exists iff* $V_x(\omega) \geq 0$ *for all* $\omega \in G$.

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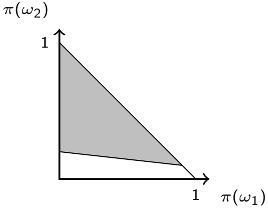


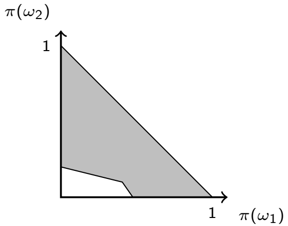
Roadmap

Empirical implications

- Empirical research eliciting the threshold for conviction (e.g., Simon & Mahan, 1971; Nagel, 1979; Dane, 1985; Connolly, 1987; Dhami, 2008):
  1. Decision-theoretic approach yields estimate of 0.50-0.60.
  2. Direct questioning yields estimate of 0.90.
- Neither predicts actual behavior particularly well.
- Different explanations have been proposed (e.g., vagueness of instructions or framing of questions).
- New explanation/conjecture based on our theory:
  1. Decision-theoretic approach:
     - Their frame is $\mathcal{G}$: we elicit p_x.
  2. Direct questioning:
     - Their frame is finer than $\mathcal{G}$: we elicit some $p_x^w \in [p_x^\ell, p_x^u]$.
     - This reflects the interpretation of the law or attitudes towards irrationalities (ex ante), but ultimately they choose rationally (ex post).
     - They prefer irrational acquittals over irrational convictions (hence 0.90 being close to p_x^u), consistently with conventional wisdom.
- More work is needed here!!!

Multinomial choice

- For $|\Gamma| > 2$, replace C_x with
$$C_\Gamma := \{\pi \in \Delta(\Omega, \mathcal{R}) : \max_{x\in\Gamma} \mathbb{E}_\pi V_x \geq 0\}.$$
- Standard of reasonable doubt "more difficult" to exist.



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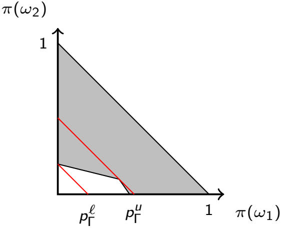
$$C_\Gamma := \{\pi \in \Delta(\Omega, \mathcal{R}) : \max_{x\in\Gamma} \mathbb{E}_\pi V_x \geq 0\}.$$

- Standard of reasonable doubt "more difficult" to exist.
- Weak standards of reasonable doubt:
  - $p^u_\Gamma := \min\{p \in [0,1] : C_\Gamma \supseteq D_p\}$ (easier to exist than p^u_x)
  - $p^\ell_\Gamma := \max\{p \in [0,1] : C_\Gamma \subseteq D_p\}$ (always exists)
- Leaving the sentence to the juror's discretion leads to lower standards (consistent with Lundberg, 2016): $p^u_\Gamma \leq p^u_x$ and $p^\ell_\Gamma \leq p^\ell_x$ for all $x \in \Gamma$.



Roadmap

Take-home messages

- Standard of reasonable doubt generically does not exist when the frame is richer than $\mathcal{G}$.
- Weak standards of reasonable doubt characterize aversion to irrationalities.
- The choice of a threshold (among the weak standards) then depends on attitudes for false negatives vs. false positives.
- Empirical observations seem to be consistent with strong aversion to irrational convictions.

Thanks for listening!!!