Pairwise epistemic conditions for Nash equilibrium

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LSE Choice Group Seminar
May 1, 2013

Roadmap

Conventional wisdom for several years

- Nash equilibrium (NE) is based on
  1. common knowledge of structure of the game,
  2. common knowledge of rationality,
  3. common knowledge of strategies being played.
- What we mean by "based on" did not become formal till Aumann & Brandenburger (1995).

Aumann & Brandenburger (1995)

- Aumann & Brandenburger (1995, ECTA; henceforth AB) were the first ones to interpret mixed strategies as conjectures and consequently a NE as a set of conjectures, rather than randomizations over actions.
- AB provided a formal set of epistemic conditions for NE:
  1. common prior,
  2. mutual belief in structure of the game,
  3. mutual belief in rationality,
  4. common belief in conjectures.
- They stressed that *common belief enters the picture in an unexpected way... what is needed is common belief of the players' conjectures, not of the players' rationality*
- They challenged the widespread view that common belief in rationality is essential for NE.

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Subsequent contributions

- Polak (1999, ECTA) noticed that AB's conditions imply common belief in rationality in complete information games.
- Polak's result restored some of the initial confidence in the importance of common belief in rationality for NE.
- Barelli (2009, GEB) generalized AB by substituting
  1. common prior with action consistency,
  2. common belief in conjectures with constant conjectures in the support of the action-consistent distribution.
- Barelli's conditions do not imply common belief in rationality, even in complete information games, thus confirming AB's initial intuition.
- His result hinges that absence of common belief in rationality from the set of epistemic conditions for NE may be attributed to the lack of a common prior.

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Global epistemic conditions

- All existing papers impose global epistemic conditions, e.g.,
  1. mutual belief in rationality:
     - everybody believes that everybody is rational.
  2. common belief in conjectures:
     - everybody believes that their conjectures are ϕ, and
     - everybody believes that everybody believes that their conjectures are ϕ, and
     - $\cdots$
- Such conditions imply pairwise epistemic conditions *for every pair* (i, j) *of players*:
  1. pairwise mutual belief in rationality for every pair (i, j):
     - both believe that both are rational.
  2. pairwise common belief in conjectures for every pair (i, j):
     - both believe that their conjectures are (ϕ_i, ϕ_j), and
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Pairwise epistemic conditions

- We impose pairwise epistemic conditions *only for some pairs of players*.
- In our main result, we substitute
  1. action-consistency with pairwise action consistency,
  2. mutual belief in the structure of the game with pairwise mutual belief in the structure of the game,
  3. mutual belief in rationality with pairwise mutual belief in rationality,
  4. constant conjectures in the support of the action-consistent distribution with pairwise constant conjectures in the pairwise action-consistent distribution.
- In our corollary, we retain the CP and we substitute
  1. mutual belief in the structure of the game with pairwise mutual belief in the structure of the game,
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Our contributions

1. We weaken not only AB's but also Barelli's conditions.
2. We show that absence of common belief in rationality from the epistemic conditions for NE should not be necessarily attributed to the lack of a common prior.
3. Our conditions do not require nor imply mutual belief in rationality, thus reinforcing AB's intuition about common belief in rationality not being crucial for NE.
4. We provide a framework for studying solution concepts from a local perspective and/or embedding the epistemic approach to the theory of networks.

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Roadmap

Normal form games

- Set of players $I = \{1, \dots, n\}$
- Player i's strategy $a_i \in A_i$
- Player i's payoff function $g_i : A_i \times A_{-i} \to \mathbb{R}$
- Player i's conjecture $\phi_i \in \Delta(A_{-i})$
- Strategy a_i is rational given ϕ_i if $a_i \in BR_i(\phi_i)$
- Player i's mixed strategy $\sigma_i \in \Delta(A_i)$
- Nash equilibrium $(\sigma_1, \dots, \sigma_n)$ if for all $i \in I$

$$a_i \in BR_i(\sigma_1 \times \cdots \times \sigma_{i-1} \times \sigma_{i+1} \times \cdots \times \sigma_n)$$

for all $a_i \in \mathrm{supp}(\sigma_i)$

Belief

- State space Ω
- Player i's information partition $\mathcal{P}_i$
- Player i's beliefs $p_i(\cdot;\omega) \in \Delta(\Omega)$ at $\omega \in \Omega$
  - Beliefs are $\mathcal{P}_i$-measurable
  - Players know their type (information set): $p_i\big(P_i(\omega);\omega\big) = 1$

Relevant events

- $\mathcal{P}_i$-measurable strategy function $\mathbf{a}_i : \Omega \to A_i$
  - $[a_i] := \{\omega \in \Omega : \mathbf{a}_i(\omega) = a_i\}$
  - $[a_{-i}] := \bigcap_{j \neq i} [a_j]$
- $\mathcal{P}_i$-measurable conjecture function $\phi_i : \Omega \to \Delta(A_{-i})$
  - $\phi_i(\omega)(a_{-i}) := p_i([a_{-i}]; \omega)$
  - $[\phi_i] := \{\omega \in \Omega : \phi_i(\omega) = \phi_i\}$
  - $[\phi_1, \ldots, \phi_n] := [\phi_1] \cap \cdots \cap [\phi_n]$
- $\mathcal{P}_i$-measurable payoff function $\mathbf{g}_i : \Omega \times A \to \mathbb{R}$
  - $[g_i] := \{\omega \in \Omega : \mathbf{g}_i(\omega, \cdot) = g_i(\cdot)\}$
  - $[g_1, \ldots, g_n] := [g_1] \cap \cdots \cap [g_n]$
- Player i is rational at ω if $\mathbf{a}_i(\omega)$ is a best response to $\phi_i(\omega)$
  - $R_i := \{\omega \in \Omega : \mathbf{a}_i(\omega) \in BR_i(\phi_i(\omega))\}$

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Common prior and action-consistency

- A probability measure $\pi \in \Delta(\Omega)$ is a common prior (CP), if for every $i \in I$ and every $\omega \in \Omega$ with $\pi\big(P_i(\omega)\big) > 0$ it is the case that $p_i(\cdot;\omega) = \pi\big(\cdot|P_i(\omega)\big)$.
- A probability measure $\mu \in \Delta(\Omega)$ is action-consistent if for every A-measurable random variable $b : \Omega \to \mathbb{R}$

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for all $i \in I$.
- An action-consistent $\mu \in \Delta(\Omega)$ exists if and only if there is no mutually beneficial action-verifiable bet (Barelli, 2009).
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Interactive belief

- Player i believes E:

$$\boxed{B_i(E) := \{\omega \in \Omega : p_i(E;\omega) = 1\}}$$

- Mutual belief in E:

$$\boxed{B(E) := B_1(E) \cap \cdots \cap B_n(E)}$$

- Common belief in E:

$$\boxed{CB(E) := B(E) \cap B(B(E)) \cap \cdots}$$

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Epistemic conditions for NE

Theorem (Aumann & Brandenburger, 1995)

Let $\pi \in \Delta(\Omega)$ be a common prior, and suppose that there is some $\omega \in supp(\pi)$ such that $\omega \in B(R_1 \cap \cdots \cap R_n) \cap B([g_1, \ldots, g_n]) \cap CB([\phi_1, \ldots, \phi_n])$. Then, there exists a mixed strategy profile $(\sigma_1, \ldots, \sigma_n)$ such that:

(i) $marg_{A_i} \phi_j = \sigma_i$ *for all* $j \in I \setminus \{i\}$,

(ii) $(\sigma_1, \ldots, \sigma_n)$ *is a Nash equilibrium.*

Theorem (Barelli, 2009)

Let $\mu \in \Delta(\Omega)$ be an action-consistent probability measure such that $(\phi_1(\omega'), \ldots, \phi_n(\omega')) = (\phi_1, \ldots, \phi_n)$ for all $\omega' \in supp(\mu)$. Moreover, assume that there is some $\omega \in supp(\mu)$ such that $\omega \in B(R_1 \cap \cdots \cap R_n) \cap B([g_1, \ldots, g_n])$. Then, there exists a mixed strategy profile $(\sigma_1, \ldots, \sigma_n)$ such that:

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Pairwise relationships

- Undirected graph $G = (I, \mathcal{E})$
  - set of players I
  - set of symmetric binary relations between players $\mathcal{E}$
- Interpretations of G
  1. pairwise relationships of purely epistemic nature
  2. physical network
- G-pairwise epistemic conditions are imposed for each $(i, j) \in \mathcal{E}$
- Global epistemic conditions are G-pairwise epistemic conditions for a complete G
- A biconnected graph remains connected after having removed an arbitrary player

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G-pairwise action consistency

- A probability measure $\mu_{i,j} \in \Delta(\Omega)$ is pairwise action-consistent between i and j if for every A-measurable random variable $b : \Omega \to \mathbb{R}$

$$\sum_{\omega \in \Omega} \mu_{i,j}(\omega) b(\omega) = \sum_{\omega \in \Omega} \mu_{i,j}(\omega) \Big(\sum_{\omega' \in P_k(\omega)} p_k(\{\omega'\}; \omega) b(\omega') \Big)$$

  for all $k \in \{i, j\}$.
- A pairwise action-consistent $\mu_{i,j} \in \Delta(\Omega)$ between i and j exists if and only if there is no mutually beneficial action-verifiable bet for i and j.
- Action consistency implies pairwise action-consistency for every pair.
- Beliefs are G-pairwise action consistent if there exists a collection $(\mu_{i,j})_{(i,j) \in \mathcal{E}}$ of probability measures with $\bigcap_{(i,j) \in \mathcal{E}} \mathrm{supp}(\mu_{i,j}) \neq \emptyset$ such that $\mu_{i,j}$ is pairwise action-consistent between i and j, for each $(i,j) \in \mathcal{E}$.

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- Beliefs are G-pairwise action consistent if there exists a collection $(\mu_{i,j})_{(i,j) \in \mathcal{E}}$ of probability measures with $\bigcap_{(i,j) \in \mathcal{E}} \mathrm{supp}(\mu_{i,j}) \neq \emptyset$ such that $\mu_{i,j}$ is pairwise action-consistent between i and j, for each $(i,j) \in \mathcal{E}$.

G-pairwise action consistency

- A probability measure $\mu_{i,j} \in \Delta(\Omega)$ is pairwise action-consistent between i and j if for every A-measurable random variable $b : \Omega \to \mathbb{R}$

$$\sum_{\omega \in \Omega} \mu_{i,j}(\omega) b(\omega) = \sum_{\omega \in \Omega} \mu_{i,j}(\omega) \Big(\sum_{\omega' \in P_k(\omega)} p_k(\{\omega'\}; \omega) b(\omega') \Big)$$

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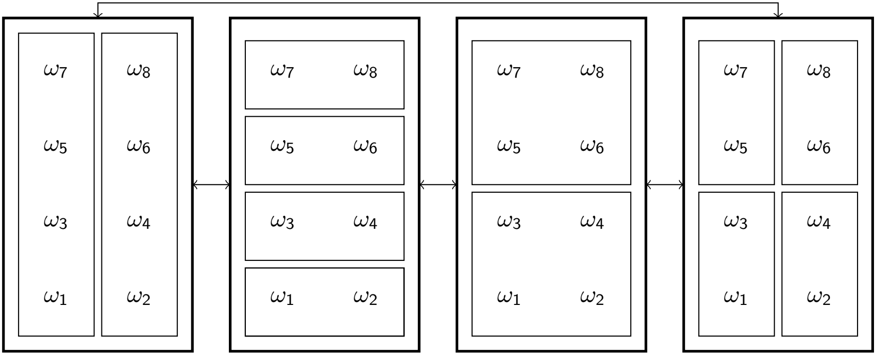

- Beliefs are G-pairwise action-consistent
  - $\mu_{A,B} = \left(\frac{1}{4} \times \omega_1; \frac{1}{4} \times \omega_3; \frac{1}{4} \times \omega_5; \frac{1}{4} \times \omega_7\right) = \mu_{A,D}$
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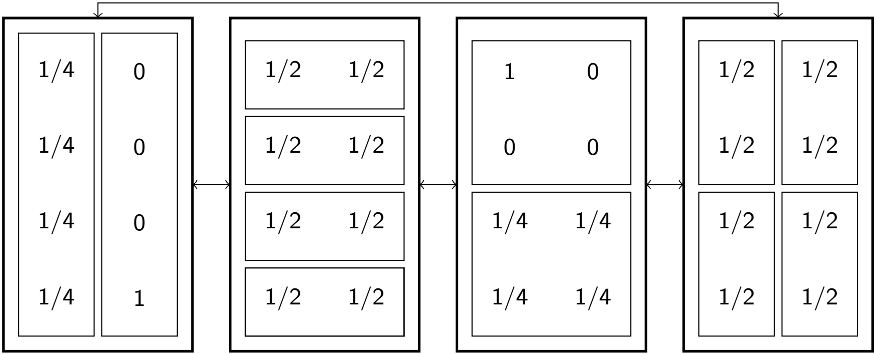

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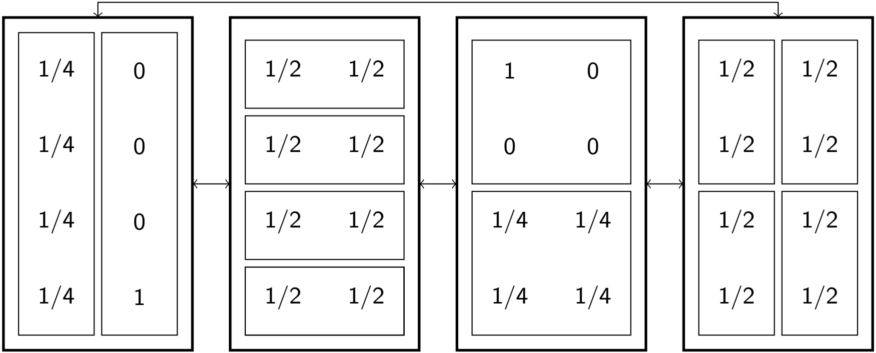

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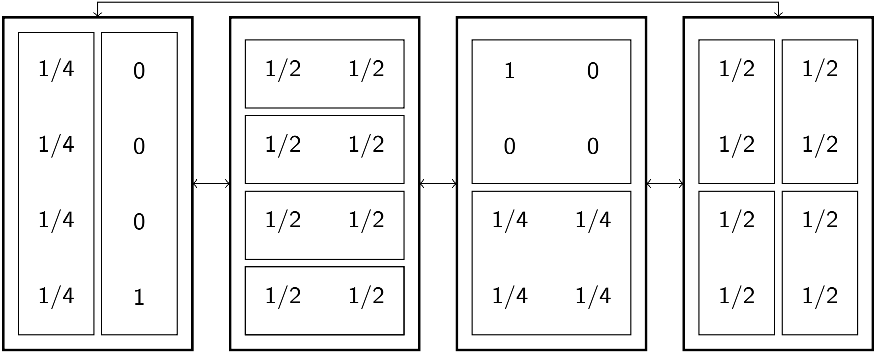

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G-pairwise constant conjectures

- If beliefs are pairwise action-consistent between i and j, the conjectures are pairwise constant in the support of the pairwise action-consistent distribution whenever $\big(\phi_i(\omega), \phi_j(\omega)\big) = (\phi_i, \phi_j)$ for all $\omega \in \text{supp}(\mu_{i,j})$.
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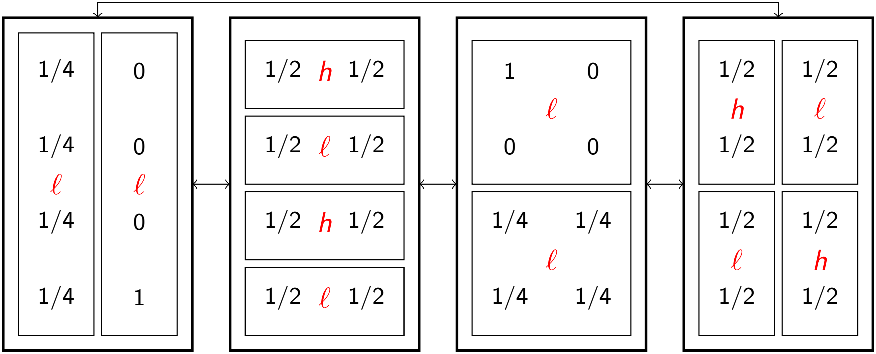

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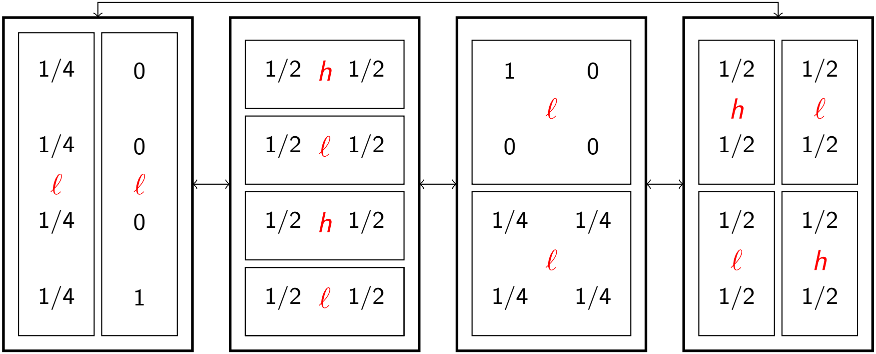

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Pairwise interactive belief

- Pairwise mutual belief between i and j:

$$\boxed{B_{i,j}(E) := B_i(E) \cap B_j(E)}$$

- Pairwise common belief between i and j:

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- Payoffs are G-pairwise mutually believed at ω whenever $\omega \in B_{i,j}([g_i] \cap [g_j])$ for all $(i,j) \in \mathcal{E}$
- G-pairwise mutual belief of payoffs is weaker than mutual belief in rationality on two dimensions

$$B([g_1, \dots, g_n]) \subseteq \bigcap_{(i,j)\in\mathcal{E}} B_{i,j}([g_i] \cap [g_j])$$

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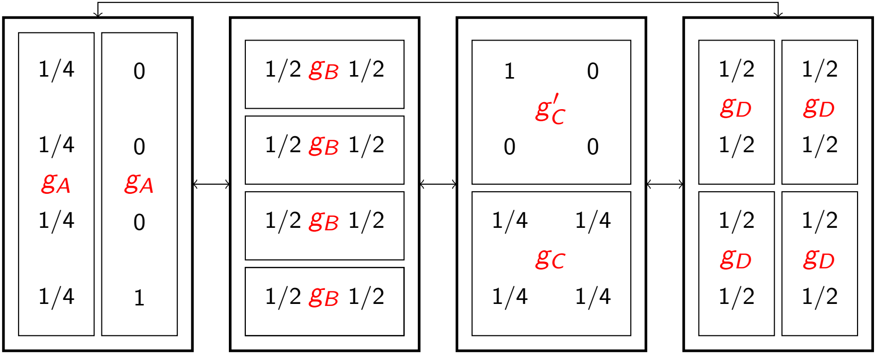

- Payoff functions are G-pairwise mutually believed at ω_1
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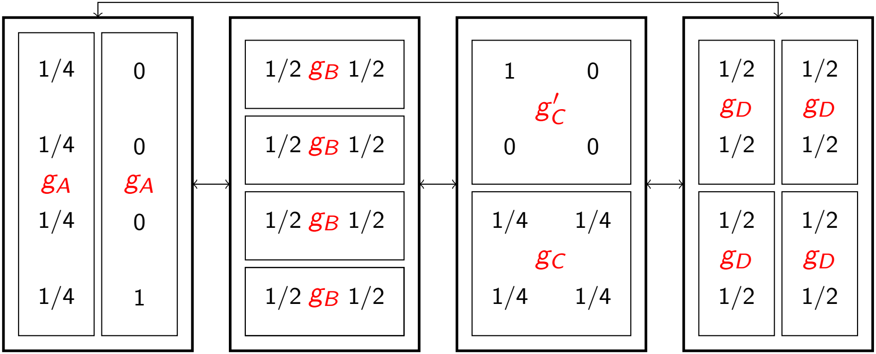

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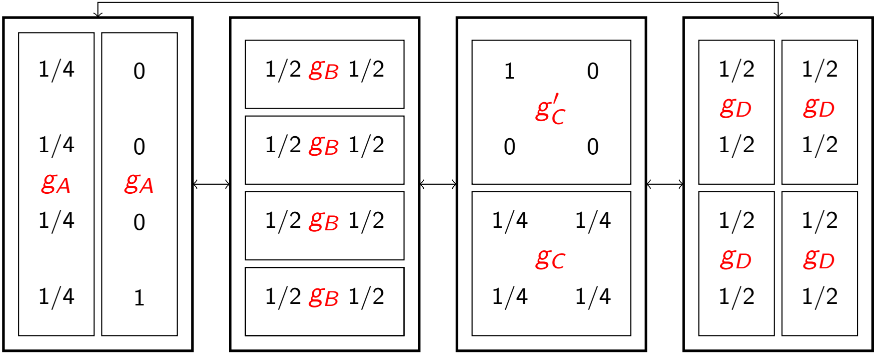

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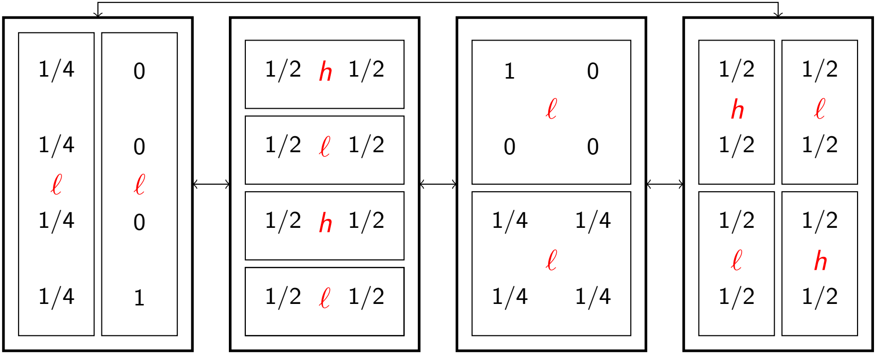

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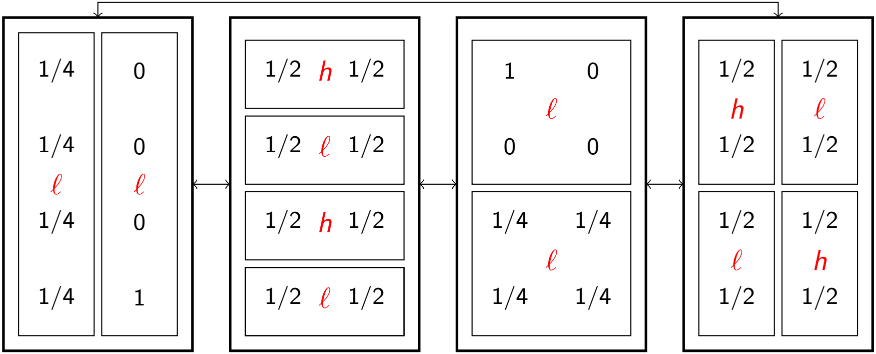

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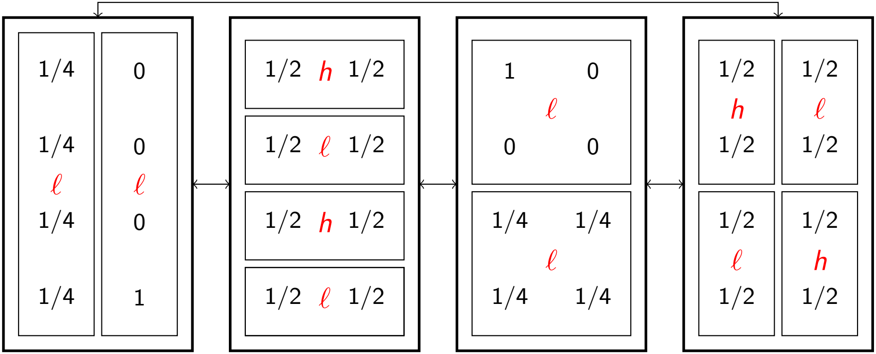

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Roadmap

Main result

Theorem

Let G be a biconnected graph and $(\phi_1, \dots, \phi_n)$ a tuple of conjectures. Suppose that for every $(i,j) \in \mathcal{E}$ there exists a pairwise action-consistent distribution $\mu_{i,j} \in \Delta(S)$ between i and j such that $\phi_k(\omega') = \phi_k$ for every $k \in \{i,j\}$ and for every $\omega' \in \mathrm{supp}(\mu_{i,j})$. Moreover, assume that there is some state $\omega \in \bigcap_{(i,j)\in\mathcal{E}} \mathrm{supp}(\mu_{i,j})$ such that $\omega \in B_{i,j}([g_i] \cap [g_j]) \cap B_{i,j}(R_i \cap R_j)$ for all $(i,j) \in \mathcal{E}$. Then, there exists a mixed strategy profile $(\sigma_1, \dots, \sigma_n)$ such that:

(i) $\mathit{marg}_{A_i} \phi_j = \sigma_i$ *for all* $j \in I \setminus \{i\}$,

(ii) $(\sigma_1, \dots, \sigma_n)$ *is a Nash equilibrium.*

Main result: sketch of the proof

- Since G is biconnected, for each $i, j, k \in I$ there exists a path connecting i and j that does not go through k. Hence, $\mathrm{marg}_{A_k}\phi_i = \mathrm{marg}_{A_k}\phi_j =: \sigma_k$.
- We show that $\phi_1(a_2, \ldots, a_n) = \phi_1(a_2) \cdots \phi_{n-1}(a_n)$. Then from the previous step it follows that $\phi_1(a_2, \ldots, a_n) = \phi_1(a_2) \cdots \phi_1(a_n)$.
- Hence, $\phi_i = \sigma_1 \times \cdots \times \sigma_{i-1} \times \sigma_{i+1} \times \cdots \times \sigma_n$.
- Finally, for every $a_i \in \mathrm{supp}(\sigma_i)$, by rationality a_i is a best response to ϕ_i and therefore to $\sigma_1 \times \cdots \times \sigma_{i-1} \times \sigma_{i+1} \times \cdots \times \sigma_n$, thus implying that $(\sigma_1, \ldots, \sigma_n)$ is a NE.

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G-pairwise common belief of conjectures

- If there exists a common prior, G-pairwise constant conjectures in the support of the common prior, coincide with G-pairwise common belief in conjectures.
- Conjectures are G-pairwise commonly believed at ω whenever $\omega \in CB_{i,j}([\phi_i] \cap [\phi_j])$ for all $(i,j) \in \mathcal{E}$
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Pairwise epistemic conditions for NE with a CP

Corollary

Let G be a biconnected graph and $(\phi_1, \ldots, \phi_n)$ a tuple of conjectures. Suppose that there exists a common prior $\pi \in \Delta(\Omega)$ and let $\omega \in supp(\pi)$ be such that $\omega \in B_{i,j}([g_i] \cap [g_j]) \cap B_{i,j}(R_i \cap R_j) \cap CB_{i,j}([\phi_i] \cap [\phi_j])$ for all $(i,j) \in \mathcal{E}$. Then, there exists a mixed strategy profile $(\sigma_1, \ldots, \sigma_n)$ such that:

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Our results in the literature

$$
\begin{array}{ccc}
\boxed{\text{Theorem}} & \Rightarrow & \boxed{\text{Barelli (2009)}} \\
\Downarrow & & \Downarrow \\
\boxed{\text{Corollary}} & \Rightarrow & \boxed{\text{Aumann \& Brandenburger (1995)}}
\end{array}
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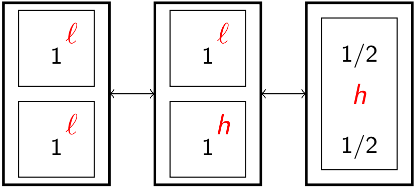
Tightness of our results: Anti-coordination game



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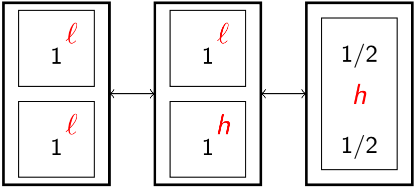

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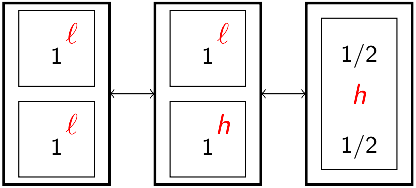

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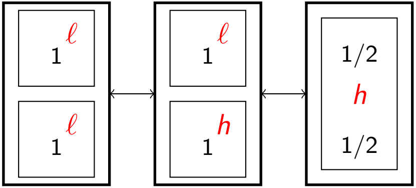

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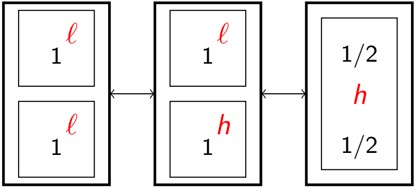

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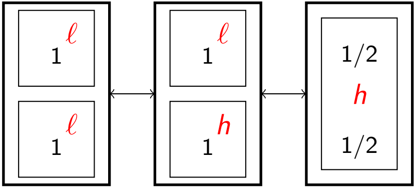

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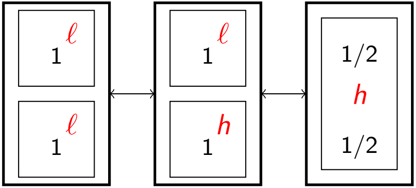
Tightness of our results: Anti-coordination game



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Contributions of this paper

1. We weaken not only AB's but also Barelli's conditions.
2. We show that absence of common belief in rationality from the epistemic conditions for NE should not be necessarily attributed to the lack of a common prior.
3. Our conditions do not require nor imply mutual belief in rationality, thus reinforcing AB's intuition about common belief in rationality not being crucial for NE.
4. We provide a framework for studying solution concepts from a local perspective and/or embedding the epistemic approach to the theory of networks.

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Thanks for listening!!!