Eliciting beliefs

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Roadmap

1. Belief elicitation
2. Problems
3. Belief updating

Roadmap

Scoring rules

Belief elicitation mechanisms

Design a mechanism to reveal beliefs through choices

1. Do not assume much about how the subject chooses
   - The subject maximizes SEU (Savage, 1954; Anscombe & Aumann, 1963)
   - Nothing assumed about utility function or the belief
2. Problem must be simple
   - At least the mechanism should be direct
   - Moreover it should be easy to explain to the subject
   - Not much data will be required
3. Problem must be incentive compatible
   - Only optimal choice to report truthfully

We seem to ask too much

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General strategy

We leverage the fact that the state is ex post observable

- The subject reports a belief
  - In principle we have no clue if the subject tells the truth
- We reward accurate reports and punish inaccurate ones
  - We need to be smart in how we implement the incentives

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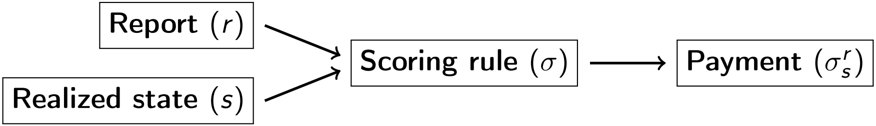
Scoring rules

- A scoring rule is the most common elicitation mechanism
- Think of it as a black box:



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- Each report can be seen as an act

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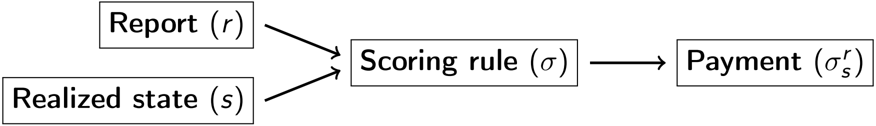

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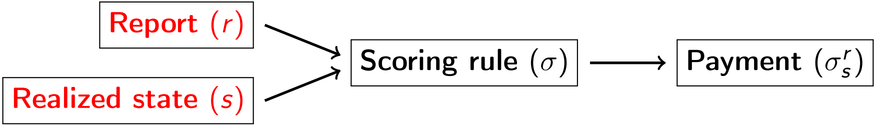

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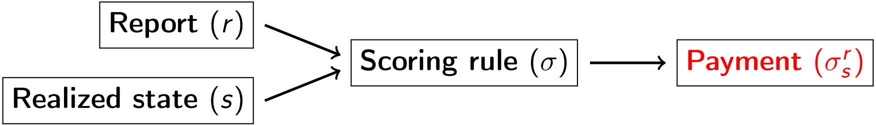

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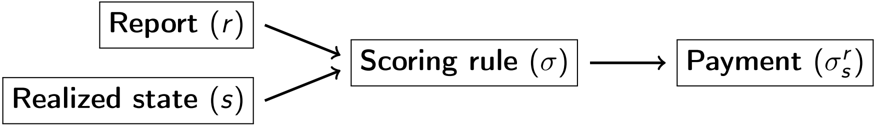


- Payment can either be monetary amount or a lottery
- Each report can be seen as an act

First attempt

Is this a good scoring rule?

	Democrats (s_1)	Republicans (s_2)
σ^r	$100 - 100(1 - r_1)$	$100 - 100(1 - r_2)$

- The SEU of a risk-neutral subject with belief μ is

$$\mathbb{E}_\mu(u(\sigma^r)) = 100\mu_1 r_1 + 100\mu_2(1 - r_1)$$

- To find the optimal report, differentiate the SEU

$$\frac{\partial \mathbb{E}_\mu(u(\sigma^r))}{\partial r_1} = 100(\mu_1 - \mu_2)$$

- Hence, the following scenarios arise:
  - If $\mu_1 > 0.5$, the optimal report is $r_1 = 0$
  - If $\mu_1 < 0.5$, the optimal report is $r_1 = 1$

Quadratic scoring rule

How about this scoring rule?

	Democrats (s_1)	Republicans (s_2)
σ^r	$100 - 100(1 - r_1)^2$	$100 - 100(1 - r_2)^2$

- The SEU of a risk-neutral subject with belief μ is

$$\mathbb{E}_\mu(u(\sigma^r)) = 100 - 100\mu_1(1 - r_1)^2 - 100\mu_2 r_1^2$$

- To find the optimal report, take the FOC

$$200\mu_1(1 - r_1) - 200\mu_2 r_1 = 0$$

- Equivalently, this yields

$$\boxed{r_1 = \mu_1}$$

meaning that the only optimal action is to report truthfully.

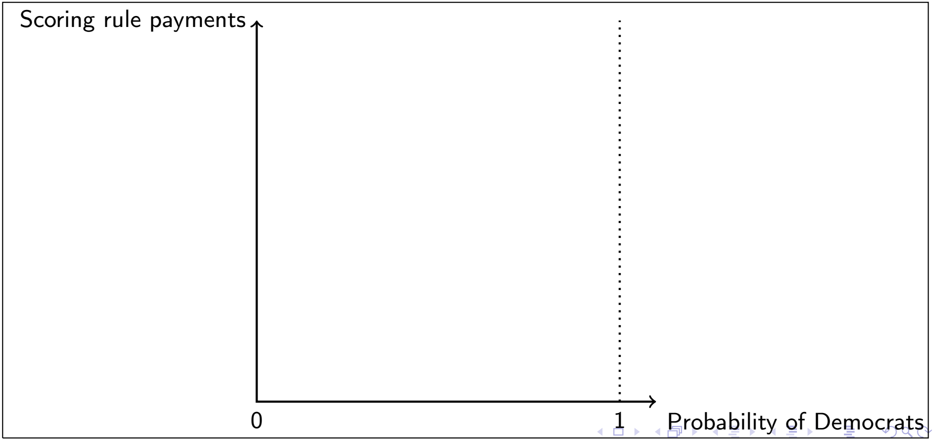
- Such a scoring rule is called (strictly) proper

Quadratic scoring rules

- Quadratic scoring rule originated in meteorology (Brier, 1950)
- Popular in many fields where they are used as incentive schemes for accuracy of forecasters
- Popular in Experimental Economics as incentive schemes for truth-telling of subjects (Schotter & Trevino, 2014)
- For the time being, put aside the fact that they assume risk neutrality

Quadratic scoring rule

	Democrats (s_1)	Republicans (s_2)
σ^r	$100 - 100(1 - r_1)^2$	$100 - 100(1 - r_2)^2$



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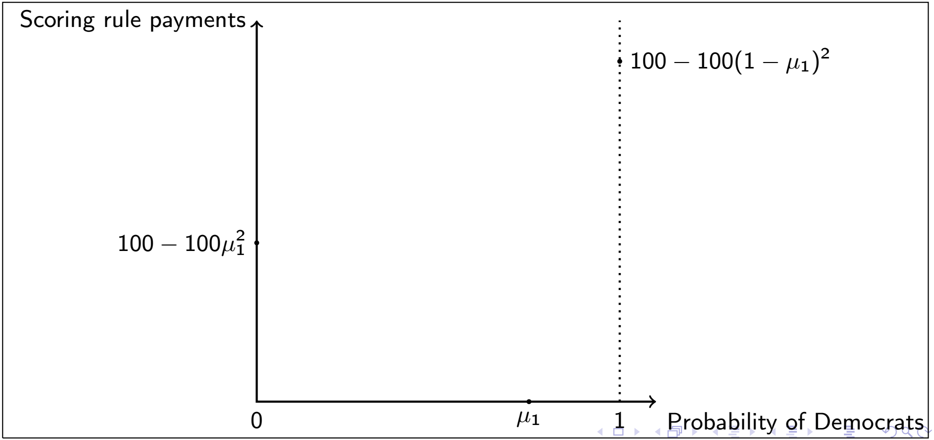

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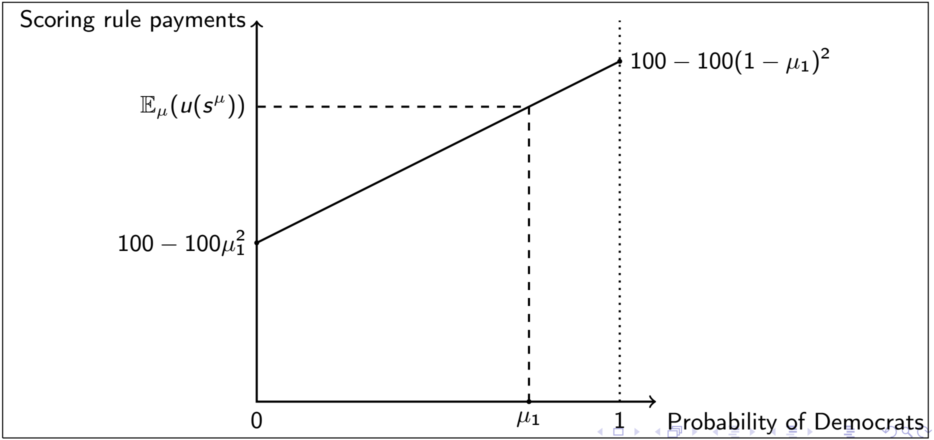

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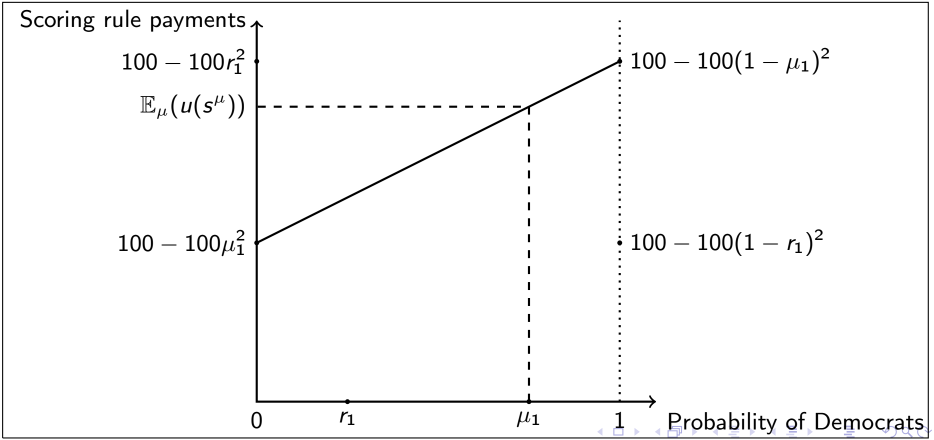

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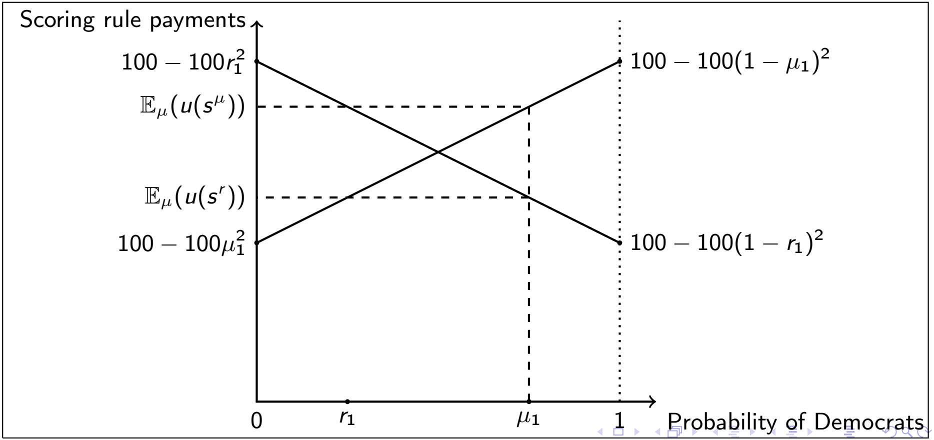

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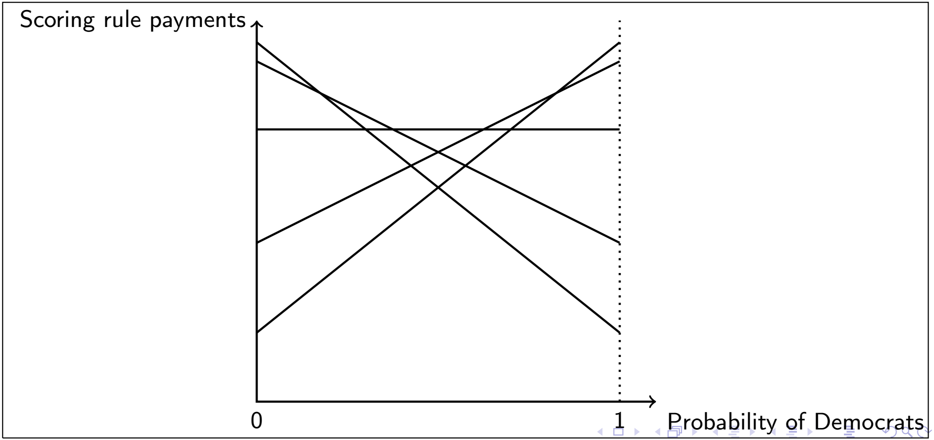

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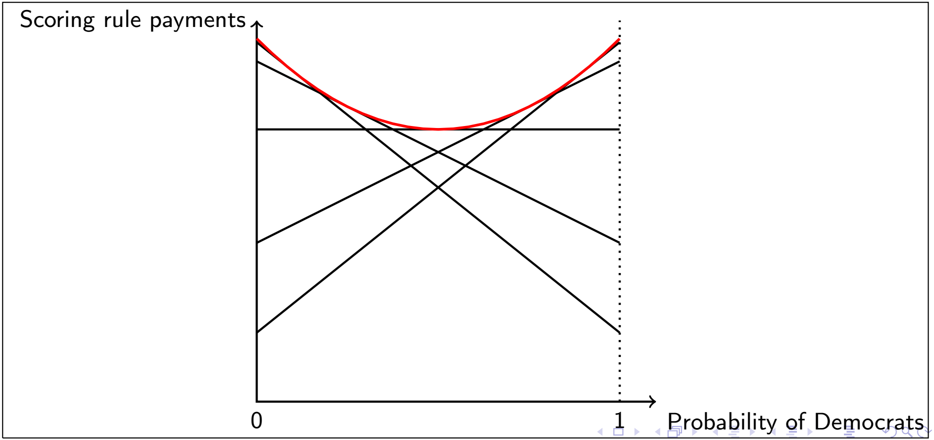

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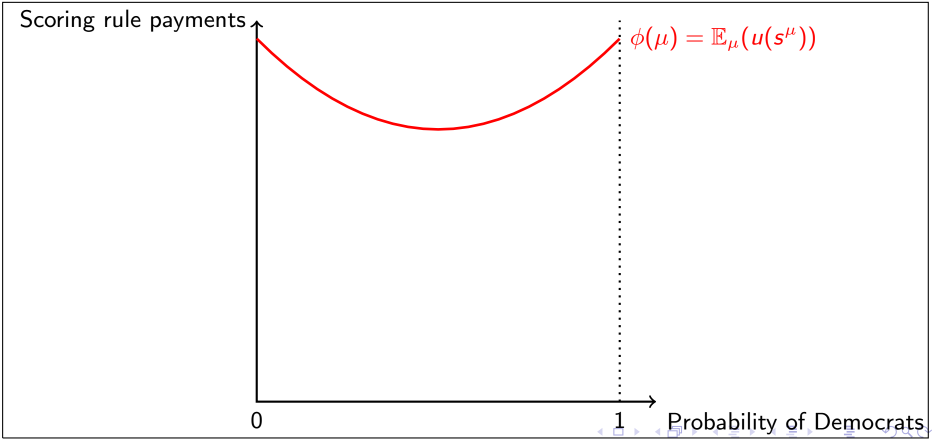

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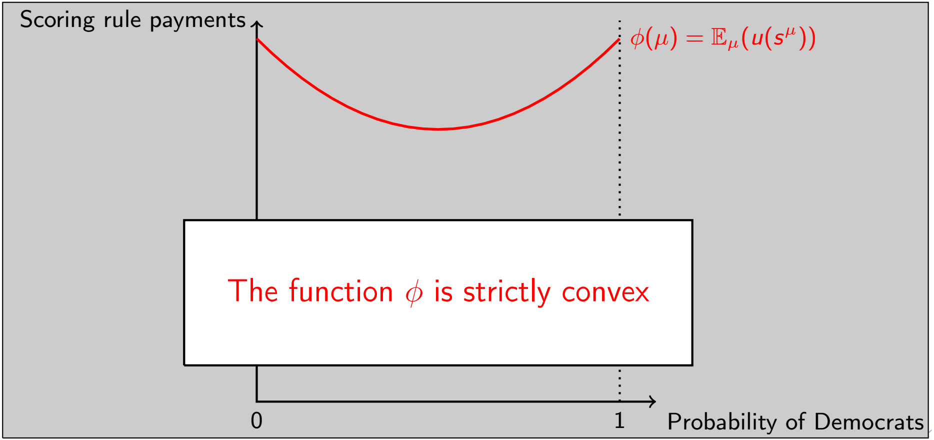
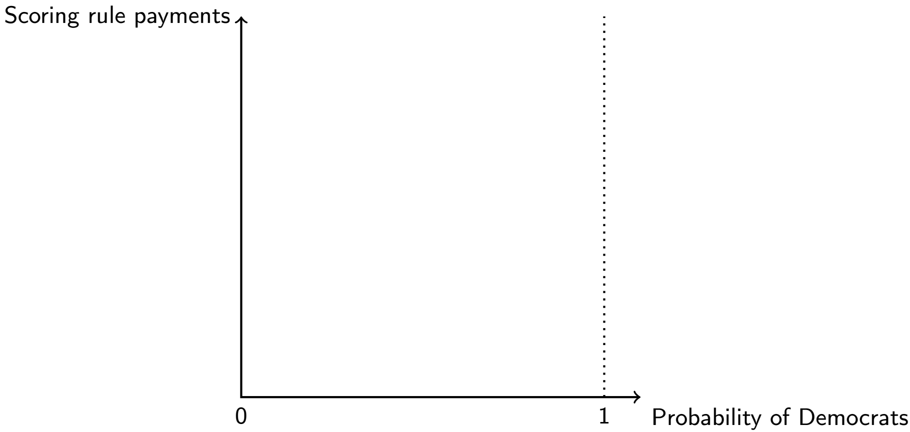


Strictly proper scoring rules



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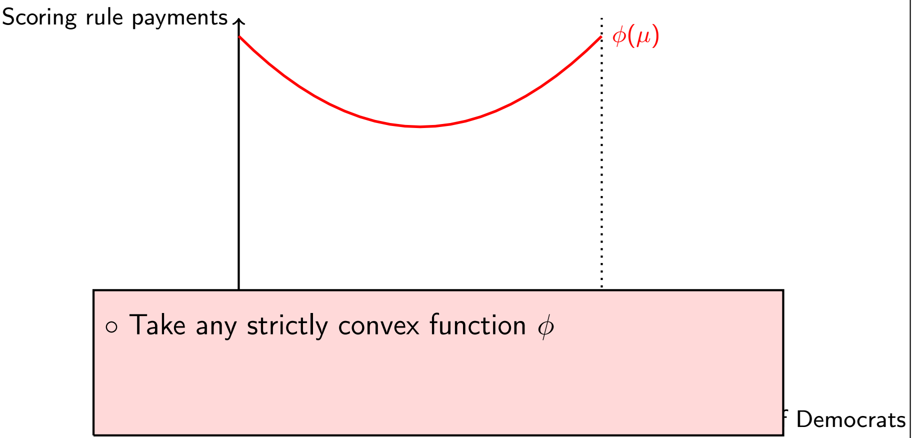
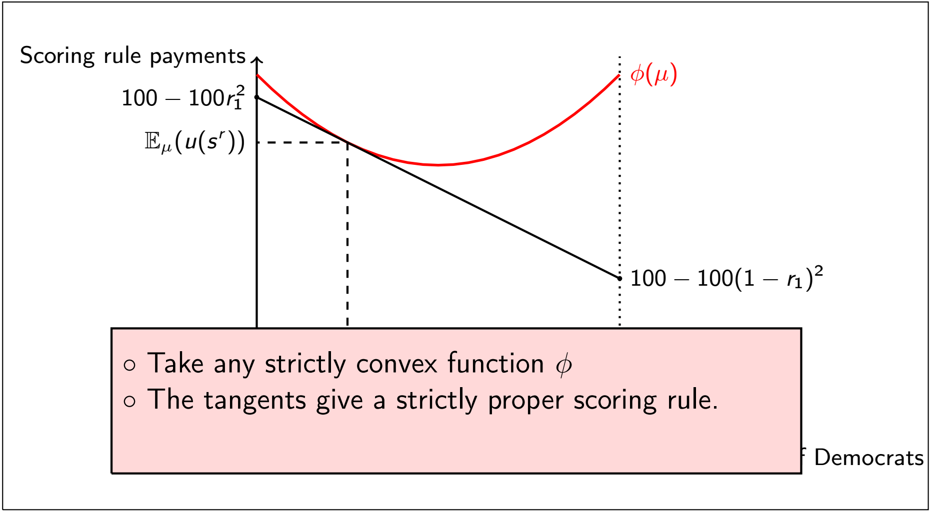


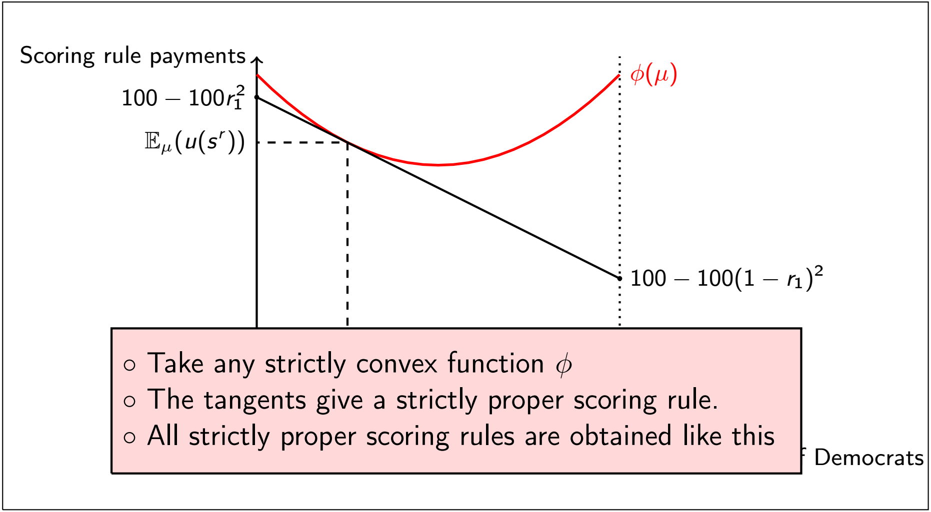
◦ Take any strictly convex function ϕ

Strictly proper scoring rules



- Take any strictly convex function ϕ
- The tangents give a strictly proper scoring rule.

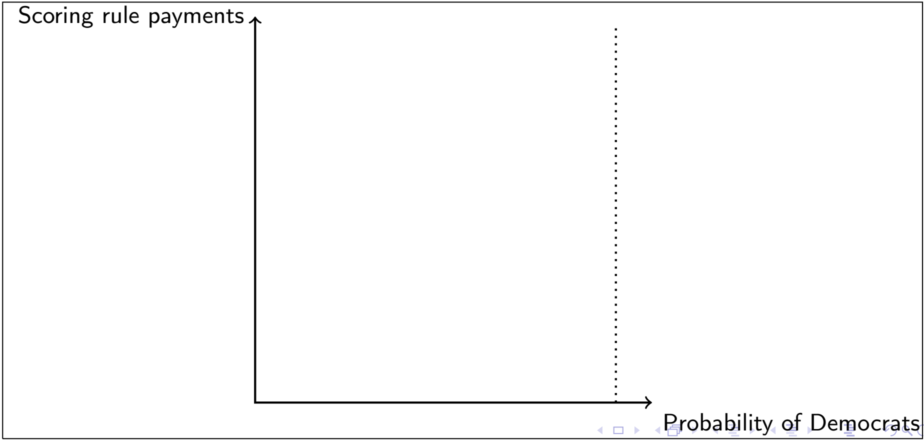
Strictly proper scoring rules



- Take any strictly convex function ϕ
- The tangents give a strictly proper scoring rule.
- All strictly proper scoring rules are obtained like this

Our first attempt revisited

	Democrats (s_1)	Republicans (s_2)
σ^r	$100 - 100(1 - r_1)$	$100 - 100r_1$



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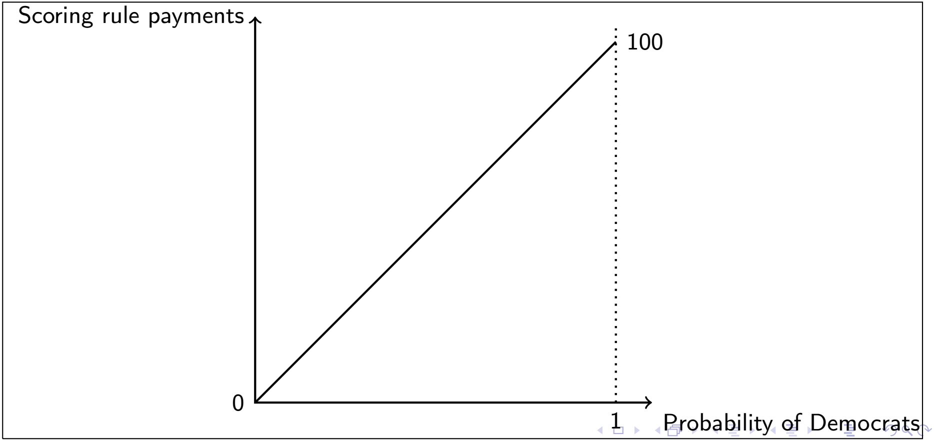

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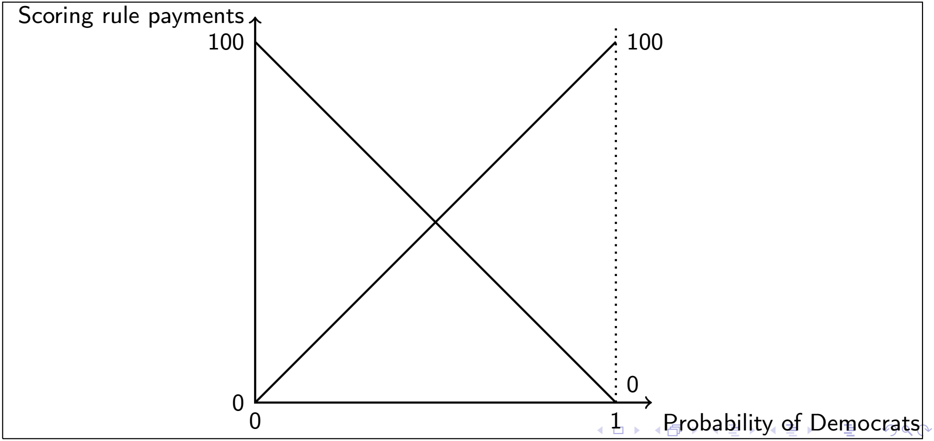

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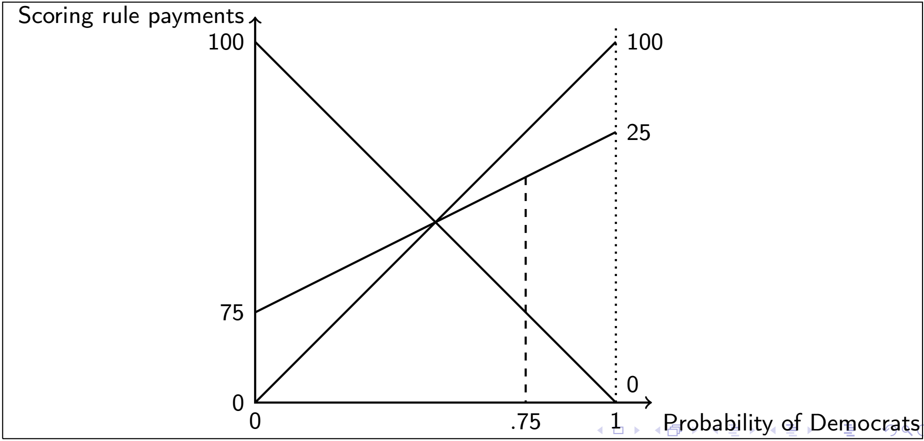

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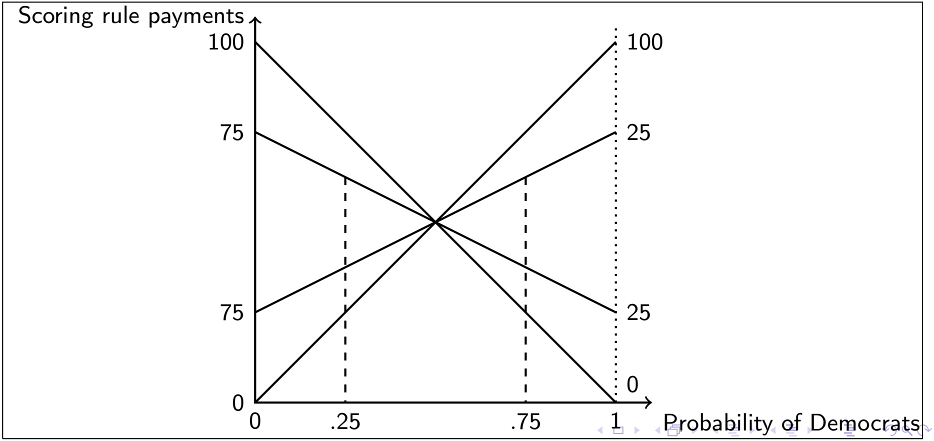

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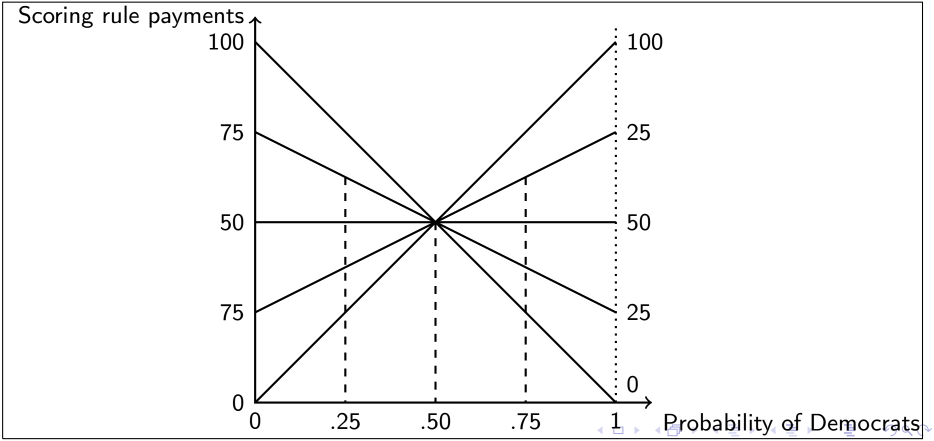

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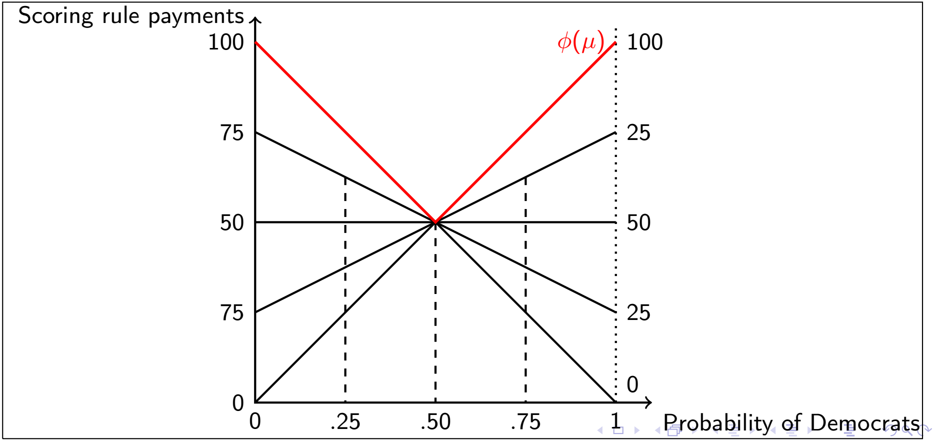
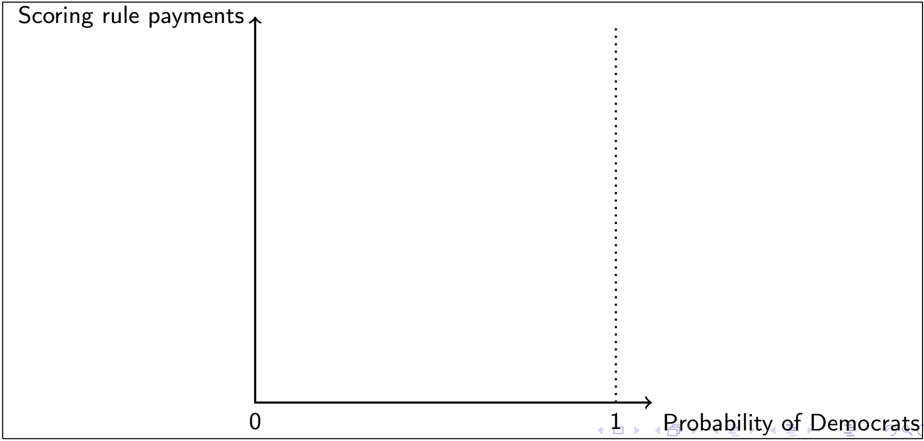


Flat reward

	Democrats (s_1)	Republicans (s_2)
σ^r	a	b



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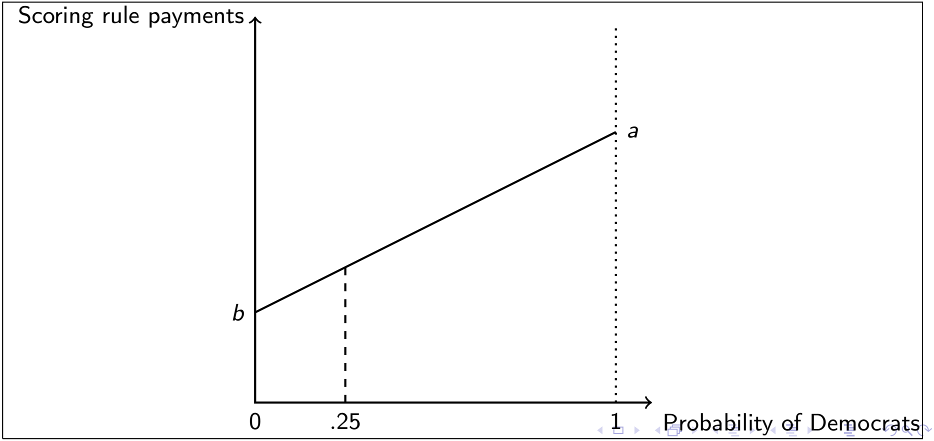

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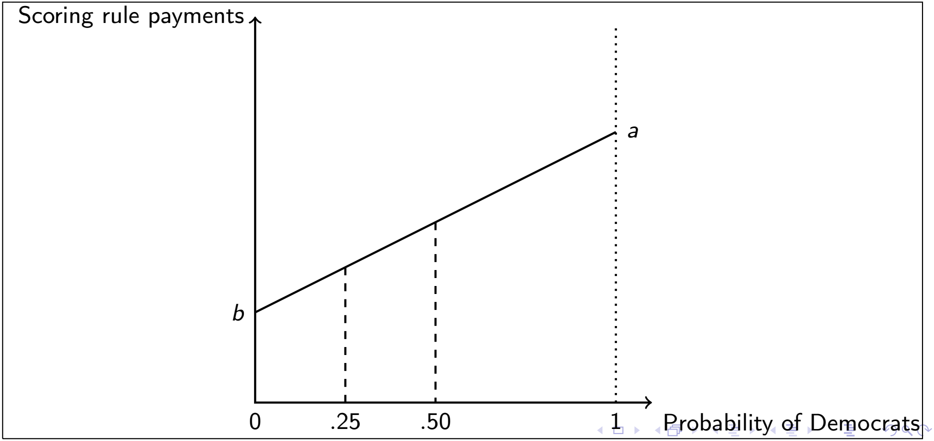

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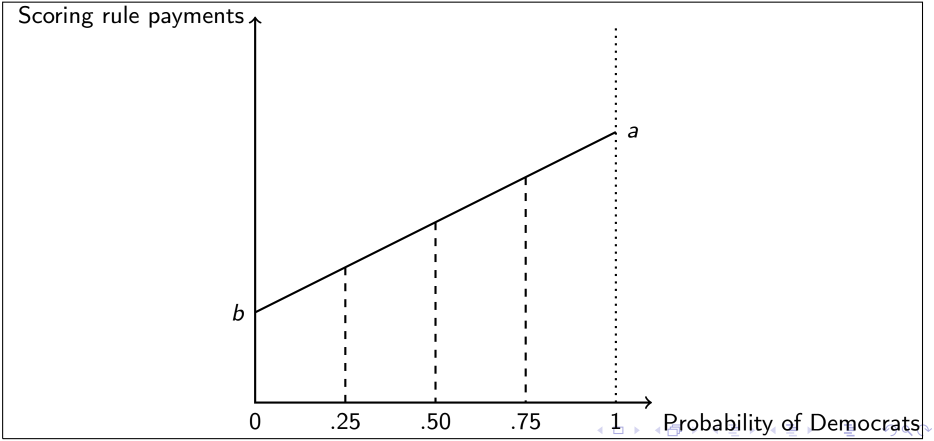

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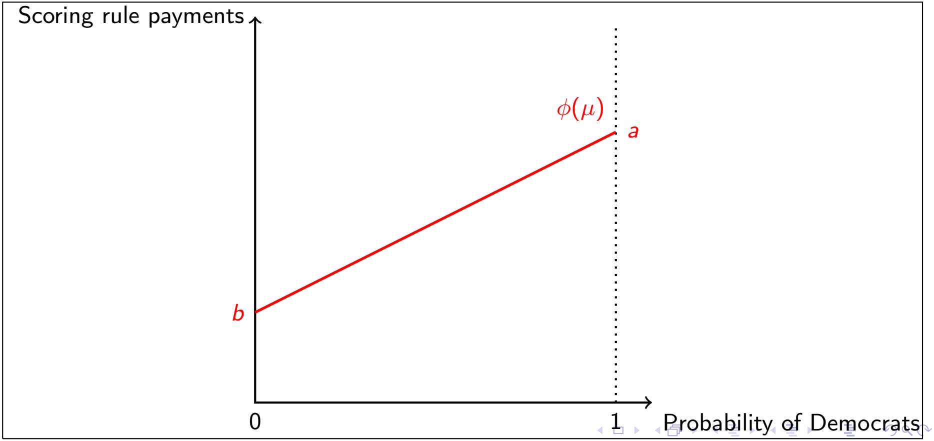


Stochastic mechanisms

Risk preferences

Is Quadratic scoring rule strictly proper
if the subject is not risk neutral?

	Democrats (s_1)	Republicans (s_2)
σ^r	$100 - 100(1 - r_1)^2$	$100 - 100(1 - r_2)^2$

- The SEU of a subject with utility function u and belief μ is

$$\mathbb{E}_\mu(u(\sigma^r)) = \mu_1 u\big(100 - 100(1 - r_1)^2\big) + \mu_2 u\big(100 - 100 r_1^2\big)$$

- The FOC becomes

$$200\mu_1(1 - r_1)u'\big(100 - 100(1 - r_1)^2\big) - 200\mu_2 r_1 u'\big(100 - 100 r_1^2\big) = 0$$

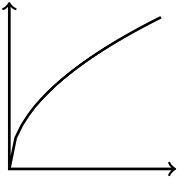
- Equivalently, this yields

$$\boxed{\frac{r_1}{1 - r_1} = \frac{\mu_1}{1 - \mu_1} \frac{u'\big(100 - 100(1 - r_1)^2\big)}{u'\big(100 - 100 r_1^2\big)}}$$

Misreporting

$$\frac{r_1}{1-r_1} = \frac{\mu_1}{1-\mu_1} \frac{u'\left(100 - 100(1-r_1)^2\right)}{u'\left(100 - 100r_1^2\right)}$$

Suppose that the optimal report is $r_1 = 0.30$

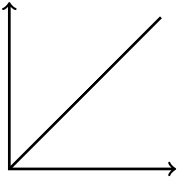


Risk averse

$\frac{u'(70)}{u'(30)} < 1$

$\mu_1 > 0.30$

(underreporting)

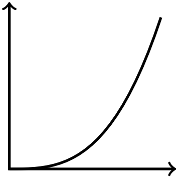


Risk neutral

$\frac{u'(70)}{u'(30)} = 1$

$\mu_1 = 0.30$

(truthtelling)



Risk seeking

$\frac{u'(70)}{u'(30)} > 1$

$\mu_1 < 0.30$

(overreporting)

General strategy

We leverage the fact that risk preferences do not matter if there are only two potential outcomes

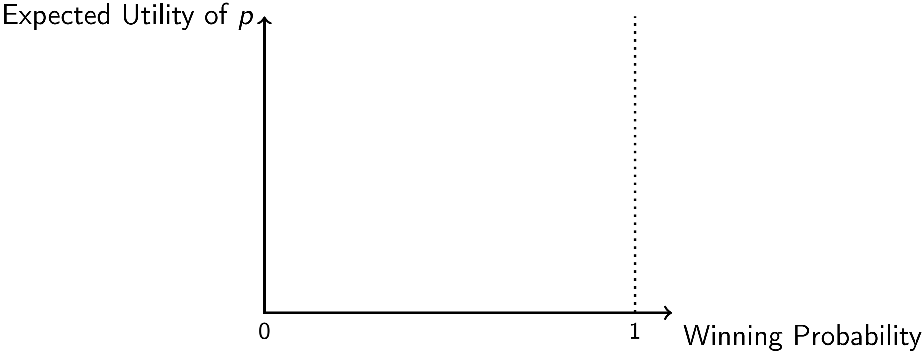
- We fix a valuable prize
  - E.g., \$100
- We pay the subject in probability (i.e., lottery tickets) to win the prize
  - E.g., instead of paying \$70, pay \$100 with 70% chance
- All payments become lotteries of the form $p = \big(\alpha \times 100, (1-\alpha) \times 0\big)$

  The payment is determined by α
- Such elicitation mechanisms are called stochastic

General strategy

Why will stochastic mechanisms work?

$$p = \big(\alpha \times 100, (1 - \alpha) \times 0\big)$$



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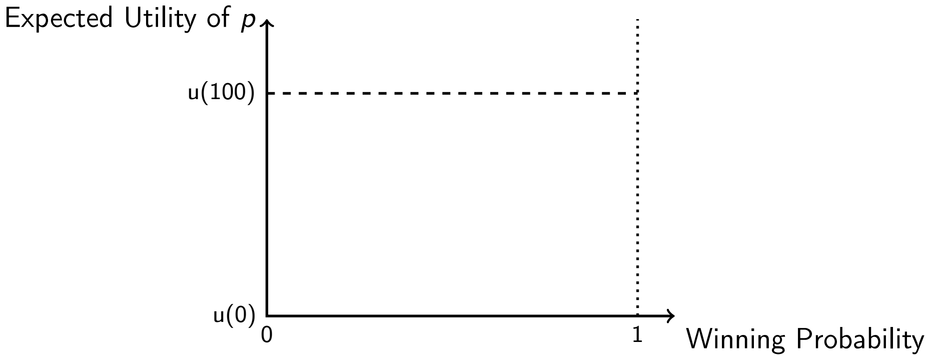

General strategy

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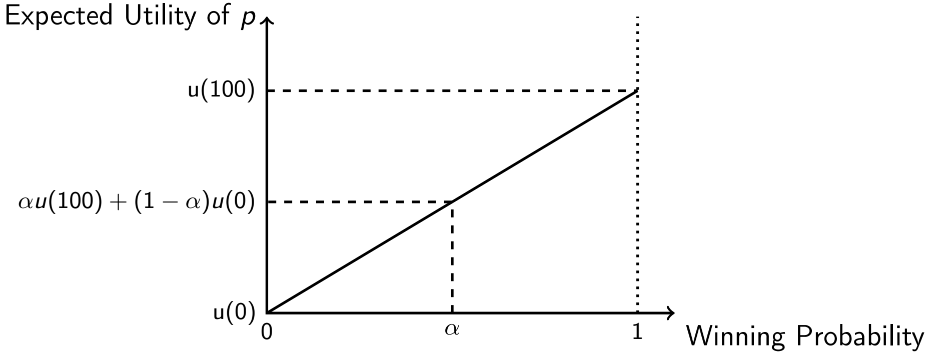

General strategy

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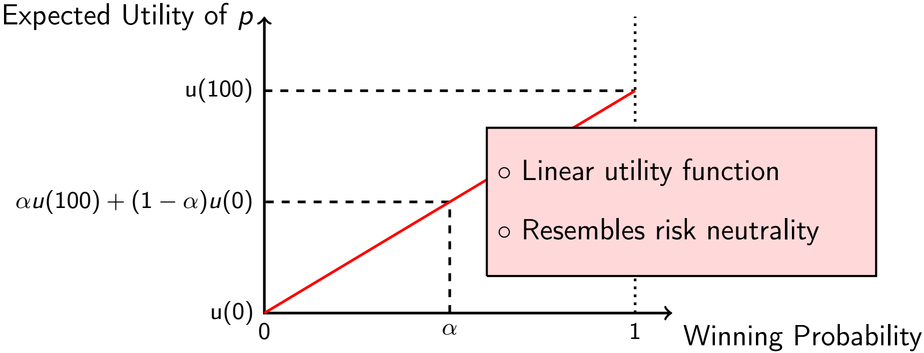


- Linear utility function
- Resembles risk neutrality

Binarized quadratic scoring rule

	Democrats (s_1)	Republicans (s_2)
σ^r	$\big(1-(1-r_1)^2\times 100, (1-r_1)^2\times 0\big)$	$\big(1-r_1^2\times 100, r_1^2\times 0\big)$

- The SEU of a risk-neutral subject with belief μ is

$$\mathbb{E}_\mu(u(\sigma^r)) = \mu_1\big(1-(1-r_1)^2\big)u(100) + \mu_2\big(1-r_1^2\big)u(100)$$

- To find the optimal report, take the FOC

$$2\mu_1(1-r_1)u(100) - 2\mu_2 r_1 u(100) = 0$$

- Equivalently, this yields

$$\boxed{r_1 = \mu_1}$$

- Only optimal action is to report truthfully, regardless of u.

Karni mechanism

- Alternative stochastic belief elicitation mechanism
- It resembles the BDM method (Becker, DeGroot & Marschak, 1964)
  - Standard method for eliciting willingness to pay
- It also looks like a second-price auction (Vickrey, 1961)
  - The most well-known strategy-proof auction

Karni mechanism

- The subject submits a report r
- We draw a random number $\alpha \in [0, 1]$
- Payments are determined as follows:
  1. If $r_1 \geq \alpha$ the subject receives the following act:

	Democrats (s_1)	Republicans (s_2)
Event	100	0

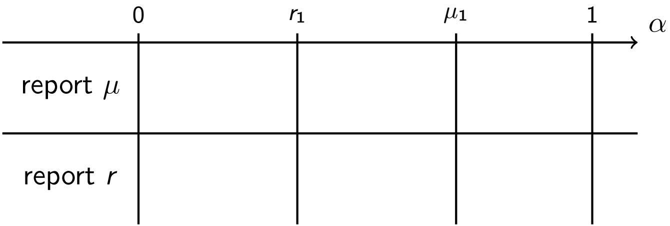
  2. If $r_1 < \alpha$ the subject receives the following lottery:

	Democrats (s_1)	Republicans (s_2)
Lottery	$(\alpha \times 100, (1-\alpha) \times 0)$	$(\alpha \times 100, (1-\alpha) \times 0)$

Karni mechanism

	Democrats (s_1)	Republicans (s_2)	SEU
Event ($r_1 \geq \alpha$)	100	0	$\mu_1 u(100)$
Lottery ($r_1 < \alpha$)	$(\alpha \times 100, (1-\alpha) \times 0)$	$(\alpha \times 100, (1-\alpha) \times 0)$	$\alpha u(100)$

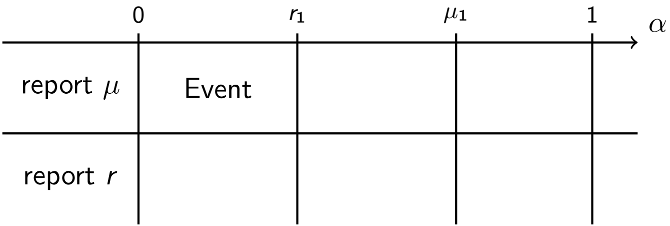
Truth telling is better than misreporting



Karni mechanism

	Democrats (s_1)	Republicans (s_2)	SEU
Event ($r_1 \geq \alpha$)	100	0	$\mu_1 u(100)$
Lottery ($r_1 < \alpha$)	$(\alpha \times 100, (1-\alpha) \times 0)$	$(\alpha \times 100, (1-\alpha) \times 0)$	$\alpha u(100)$

Truth telling is better than misreporting



Karni mechanism

	Democrats (s_1)	Republicans (s_2)	SEU
Event ($r_1 \geq \alpha$)	100	0	$\mu_1 u(100)$
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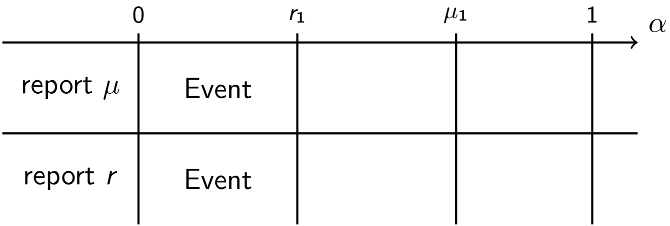

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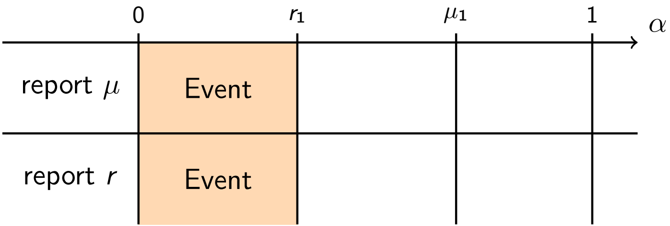

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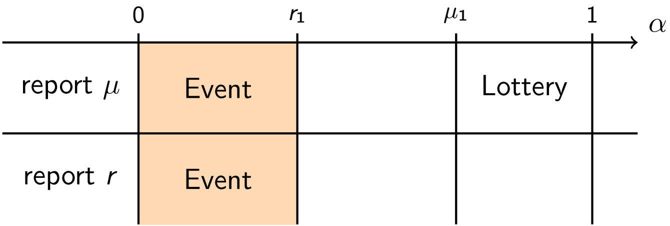

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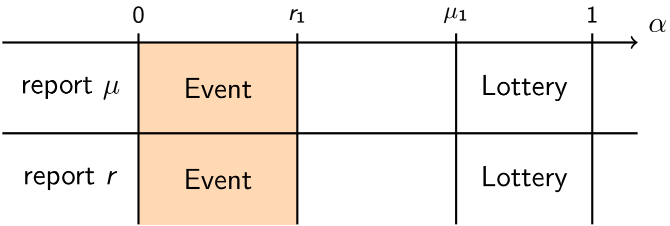

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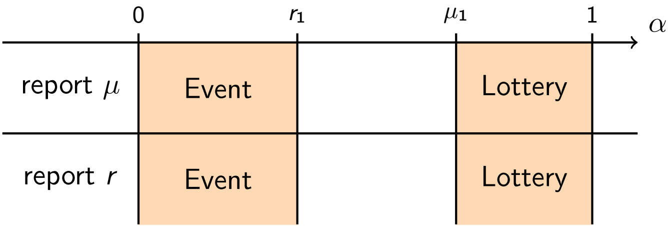

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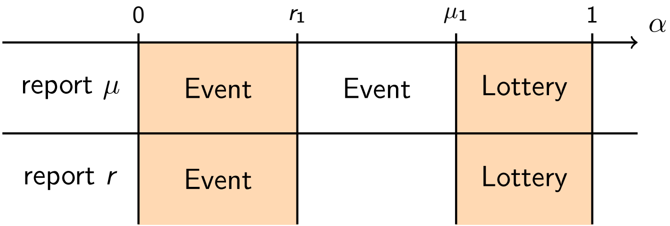

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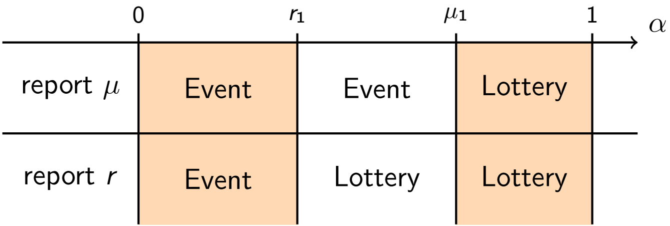

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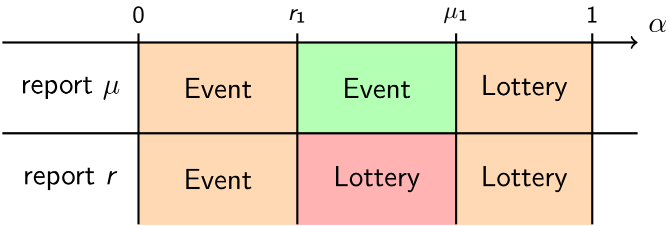


Roadmap

Hedging

Testing rationality

Underlying decision problem

	Democrats (s_1)	Republicans (s_2)
Bet on Democrats (a_1)	100	0
Bet on Republicans (a_2)	0	100

Belief elicitation mechanism

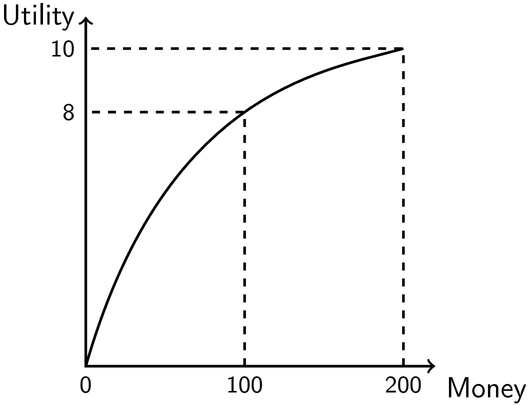
	Democrats (s_1)	Republicans (s_2)
σ^r	$\Big((1-(1-r_1)^2)\times 100, (1-r_1)^2\times 0\Big)$	$\Big(1-r_1^2\times 100, r_1^2\times 0\Big)$

Conclude that subject is rational if one of the following holds:

1. Rationally bets on Democrats: Chooses a_1 and reports $r_1 \geq 50\%$
2. Rationally bets on Republicans: Chooses a_2 and reports $r_1 \leq 50\%$

Experimental subject

- Actual belief is $\mu_1 = 0.75$ (should rationally bet on Democrats)
- The utility function looks as follows



None of this should matter: Binarized scoring rule

Experimental subject bets on Democrats

	Democrats (s_1)	Republicans (s_2)
$a_1 + \sigma^r$	$\big((1-(1-r_1)^2)\times 200, (1-r_1)^2\times 100\big)$	$\big((1-r_1^2)\times 100, r_1^2\times 0\big)$

- The subject's SEU:

$$\mathbb{E}_\mu(u(a_1+\sigma^r)) = \mu_1\big(1-(1-r_1)^2\big)u(200) + \mu_1(1-r_1)^2 u(100) + \mu_2(1-r_1^2)u(100)$$

$$= \frac{30}{4}\big(1-(1-r_1)^2\big) + 6(1-r_1)^2 + 2(1-r_1^2)$$

- The best report combined with a_1 is given by FOC:

$$15(1-r_1) - 12(1-r_1) - 4r_1 = 0$$

- So, together with a_1, the subject reports:

$$r_1 = 3/7$$

- The overall Expected Utility is approximately 8.64

Experimental subject bets on Republicans

	Democrats (s_1)	Republicans (s_2)
$a_2 + \sigma^r$	$\big((1-(1-r_1)^2)\times 100, (1-r_1)^2\times 0\big)$	$\big((1-r_1^2)\times 200, r_1^2\times 100\big)$

- The subject's SEU:

$$\begin{aligned}\mathbb{E}_\mu(u(a_2+\sigma^r)) &= \mu_1\big(1-(1-r_1)^2\big)u(100)+\mu_2(1-r_1^2)u(200)+\mu_2 r_1^2 u(100)\\ &= 6\big(1-(1-r_1)^2\big)+\frac{10}{4}(1-r_1^2)+2r_1^2\end{aligned}$$

- The best report combined with a_2 is given by FOC:

$$12(1-r_1)-5r_1+4r_1=0$$

- So, together with a_2, the subject reports:

$$r_1 = 12/13$$

- The overall Expected Utility is approximately 8.04

Putting it all together

- The true belief is $\mu_1 = 3/4$ while the reported belief is $r_1 = 3/7$
- The optimal action is to bet on the Democrats
- The subject seems to be irrational (false conclusion)
- The reason is that we do not take into account the hedging opportunity (Blanco et al., 2010)

Resolving the hedging problem

- In the lab, we randomly pay one of the two tasks:
  - Either the decision problem or the belief elicitation task
  - Definitely not both of them
- This way we disentangle the two tasks
- This is common practice in experimental economics in general
- In the field, the problem persists:
  - We cannot control side payments (e.g., portfolio of an investor)
  - We come back to this point later

State-dependent utilities

Belief identification problem

	Democrats (s_1)	Republicans (s_2)
σ^r	$\big(1-(1-r_1)^2 \times 100, (1-r_1)^2 \times 0\big)$	$\big(1-r_1^2 \times 100, r_1^2 \times 0\big)$

$$\mathbb{E}_\mu(u(\sigma^r)) = \mu_1\big(1-(1-r_1)^2\big)u_1(100) + \mu_2\big(1-r_1^2\big)u_2(100)$$

- Implicitly assumed that subject does not care about election winner
- Consider instead a Republican partizan (valuing money more at s_2):

$$\boxed{u_1(100) < u_2(100)}$$

- The following SEU model induces the same data, but different beliefs:

$$\mathbb{E}_\mu(u(\sigma^r)) = \bar{\mu}_1\big(1-(1-r_1)^2\big)\underbrace{\frac{\mu_1}{\bar{\mu}_1}u_1(100)}_{u(100)} + \bar{\mu}_2\big(1-r_1^2\big)\underbrace{\frac{\mu_2}{\bar{\mu}_2}u_2(100)}_{u(100)}$$

- Binarized scoring rule elicits $\bar{\mu}$ instead of μ

Belief identification problem

> *"the problem is serious, but I am willing to live with it until something better comes along"*
>
> *Leonard J. Savage (1971)*
> *letter correspondence with Bob Aumann*

Go beyond traditional betting data:

- Dréze (1961): agent can influence the state realization
- Fishburn (1973); Karni (1992, 1993): agent makes choices conditional on different events
- Karni, Schmeidler & Vind (1983): choices given hypothetical beliefs
- Schervish, Seidenfeld & Kadane (1990): agent compares lotteries at different states
- ...

Belief identification by proxy (Tsakas, 2025)

$$S = \{\text{Democrat } (s_1),\ \text{Republican } (s_2)\}$$
$$T = \{\text{expert } (t_1),\ \text{charlatan } (t_2)\}$$

Belief identification by proxy (Tsakas, 2025)

$$S = \{\text{Democrat } (s_1),\ \text{Republican } (s_2)\}$$
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(P_0) Given outcome, irrelevant where the opinion came from
(P_1) Known chances of opinion coming from expert
(P_2) Charlatan's opinion is uninformative
(P_3) Expert's opinion contains information

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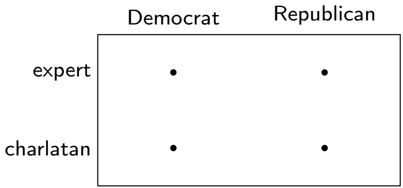

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	Democrat	Republican
expert	0.75	0.25
charlatan	0.25	0.75

The conditional beliefs are uniquely identified with standard tools.

Belief identification by proxy (Tsakas, 2025)

$$S = \{\text{Democrat } (s_1), \text{ Republican } (s_2)\}$$
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	Democrat	Republican
expert (0.55)	0.75	0.25
charlatan (0.45)	0.25	0.75

Belief identification by proxy (Tsakas, 2025)

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	Democrat	Republican
expert (0.55)	0.75	0.25
charlatan (0.45)	0.25	0.75

Equation $0.75\pi_S(\text{Democrat}) + 0.25\pi_S(\text{Republican}) = 0.55$ has unique solution

Belief identification by proxy (Tsakas, 2025)

$$S = \{\text{Democrat } (s_1), \text{ Republican } (s_2)\}$$
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(P_3) Expert's opinion contains information

	Democrat	Republican
expert	0.45	0.10
charlatan	0.15	0.30

Joint belief π is identified

Belief identification by proxy (Tsakas, 2025)

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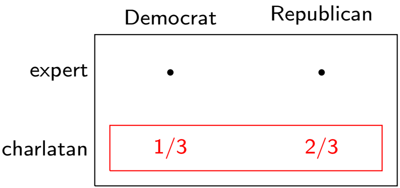


Conditional belief with respect to E

Unobservable state

Truth serums

- All mechanisms rely on being able in the end to observe the state realization

  This is not always feasible:
  - Probability that OJ Simpson was guilty
  - Probability global warming destroys us by 2200
  - ...

- Truth serums are mechanisms that deal with such problems (Prelec, 2004)

- Main idea is to elicit:
  - Beliefs about underlying event
  - Second order beliefs

- Incentives designed so that it is (BN) Equilibrium to tell the truth

Roadmap

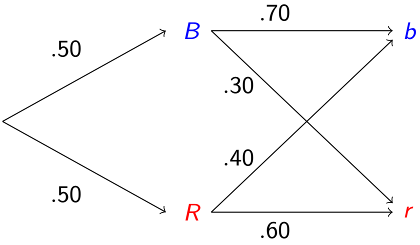
Updating biases

- Belief updating is the change in beliefs after new information has arrived
- The rational way is to apply Bayesian updating
- Evidence suggests that people do not update rationally
- We want to understand what sort of mistakes they make

Updating experiments in the lab

- There are two urns:
  - Blue urn containing 7 blue and 3 red balls
  - Red urn containing 4 blue and 6 red balls
- Randomly pick one of the two urns, and draw a random ball from it
- Tell the subject the color of the ball you drew
- Ask the subject the probability of the ball coming from the blue urn

Updating experiments in the lab

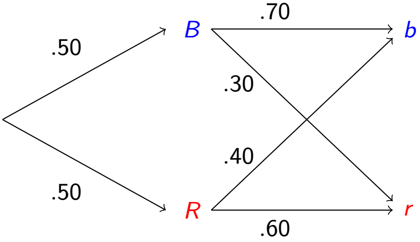


- Suppose a blue ball is drawn
- Bayesian update is given by

$$\pi(B|b) = \frac{\pi(B)\pi(b|B)}{\pi(B)\pi(b|B) + \pi(R)\pi(b|R)} = \frac{7}{11}$$

- When a posterior belief different than 7/11 is reported, we want to know what sort of mistake has been made.

Grether updating model

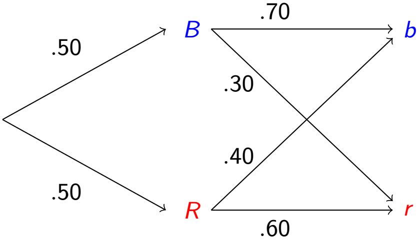


- Two types of mistake: inference mistake and base rate misuse
- Updating is carried out using the following formula:

$$\pi(B|b) = \frac{\pi(B)^{c}\pi(b|B)^{d}}{\pi(B)^{c}\pi(b|B)^{d} + \pi(R)^{c}\pi(b|R)^{d}}$$

- Parameter $c > 0$ describes base rate mistakes
- Parameter $d > 0$ describes inference mistakes

Grether updating model



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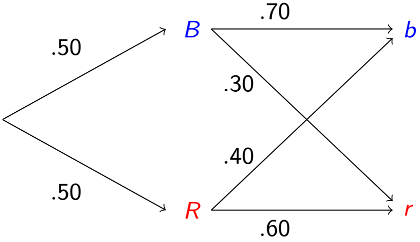

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Grether updating model

$$\pi(B|b) = \frac{\pi(B)^{c}\pi(b|B)^{d}}{\pi(B)^{c}\pi(b|B)^{d} + \pi(R)^{c}\pi(b|R)^{d}}$$

After $\pi(B|b)$ is reported, we obtain the following equation:

$$\frac{\pi(B|b)}{\pi(R|b)} = \left(\frac{\pi(B)}{\pi(R)}\right)^{c} + \left(\frac{\pi(b|B)}{\pi(b|R)}\right)^{d}$$

Taking logarithms on both sides:

$$\underbrace{\log\left(\frac{\pi(B|b)}{\pi(R|b)}\right)}_{\text{Log posterior odds}} = c\underbrace{\log\left(\frac{\pi(B)}{\pi(R)}\right)}_{\text{Log prior odds}} + d\underbrace{\log\left(\frac{\pi(b|B)}{\pi(b|R)}\right)}_{\text{LLR}}$$

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	< 1	$= 1$	> 1
Estimated $\hat{c}$	Base rate neglect	Bayesian	Base rate overuse
Estimated $\hat{d}$	Underinference	Bayesian	Overinference

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Thanks for listening!!!